PHY 104

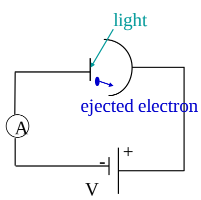
COURSE OUTLINE

Photoelectric emission/effect and thermionic emission,

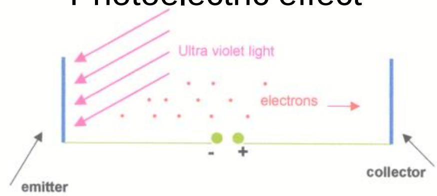
Compton effect, quantum theory of light, photons as particles, matter waves, de Broglie hypothesis, momentum and energy of photons, wave-particle duality. Heisenberg's uncertainty principle.

Photoelectric Effect

Light hits a metal plate, and electrons are ejected. These electrons are collected in the circuit and form a current.



Photoelectric effect

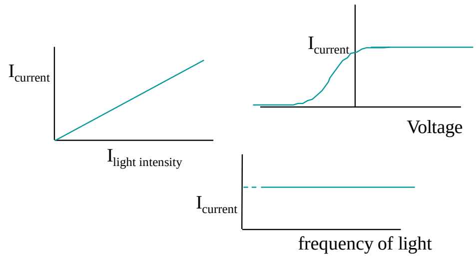


- It was found that no electrons were emitted unless the frequency of light was above a certain value, called the **threshold frequency**.
- Electrons were emitted immediately … there was no delay after first exposure.

PDST Resources for Leaving Certificate Physics

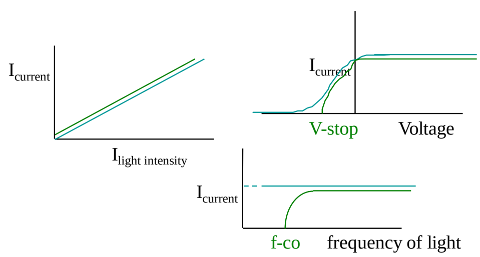
Photoelectric Effect

The following graphs illustrate what the **wave theory** predicts will happen:



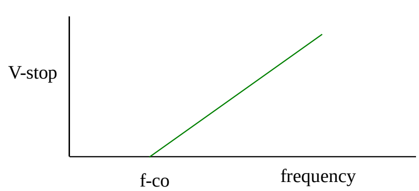
Photoelectric Effect

We now show in green what **actually happens**:



Photoelectric Effect

In addition, we see a **connection** between the stopping voltage and frequency above f-co:



Photoelectric Effect

- Einstein received the Nobel Prize for his explanation of this. (He did NOT receive the prize for his theory of relativity.)

Photoelectric Effect

- Einstein suggested that light consisted of discrete units of energy, **$\Delta E = hf$**. Electrons could either get hit with and absorb a whole photon, or they could not. There was no in-between (getting part of a photon).
- If the energy of the unit of light (photon) was not large enough to let the electron escape from the metal, no electrons would be ejected. Hence, the existence of f-co. (f-co = frequency cutoff)

Photoelectric Effect

If the photon energy were large enough to eject the electron from the metal (here, W is the energy necessary to eject the electron called the Work Function), then the following equation would apply: **$hf = W + KE$**

The energy of the photon absorbed (hf) goes into **ejecting the electron (W)** plus any extra energy left over which would show up as kinetic energy (KE).

Photoelectric Effect

- This extra kinetic energy (KE) would allow the electron to climb up a “hill”, but the size of the hill that the electron could climb up would be limited to the extra kinetic energy the electron had. By measuring the steepest hill, we could arrive at the extra energy of the electron.
- Hill sizes in electrical terms are in VOLTS: $KE = PE = qV_{stop}$.

Photoelectric Effect

Put into a nice equation:

- **$hf = W + e*V_{stop}$**
  - where f is the frequency of the light
  - W is the “WORK FUNCTION”, or the amount of energy needed to get the electron out of the metal
  - V_{stop} is the stopping potential
- When $V_{stop} = 0$, $f = f_{cutoff}$, and $hf_{cutoff} = W$.
- STOPPING POTENTIAL is the potential at which the current of the material decreases to zero
- THRESHOLD FREQUENCY is the frequency minimum frequency that produce photoelectron emission instantaneously from a given substance. Thus threshold frequency can also be define as

$$hf_{cutoff} = W$$

Photoelectric Effect - Example

- Most metals have a work function on the order of several electron volts. Copper has a work function of 4.5 eV.
- Therefore, the cut-off frequency for light ejecting electrons from copper is:

$hf_{cutoff} = 4.5$ eV, or

$f_{cutoff} = 4.5 \times (1.6 \times 10^{-19}\ C) \times (1\ V) / 6.63 \times 10^{-34}$ J-sec

$= 1.09 \times 10^{15}$ Hz,

Photoelectric Effect - Example

- or λ_{cutoff} = c/ f_{cutoff} , or

λ_{cutoff} = $(3 \times 10^8$ m/s) / $(1.09 \times 10^{15}$ cycles/sec)

= 276 nm (in the UV range)

- Any frequency lower than the cut-off (or any wavelength greater than the cut-off value) will NOT eject electrons from the metal.

Photoelectric Effect

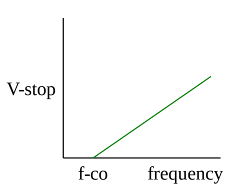
From Einstein's equation: **$hf = W + e^*V_{stop}$** , we can see that the straight line of the **V_{stop} vs f** graph should [after re-writing the equation as **$V_{stop} = (h/e)f - W/e$**] have a **slope of (h/e)** . This gives a second way of determining the value of h. [The first was from fitting the blackbody curve.] When we do this, we get the same value for h that Planck did: **6.63×10^{-34} Joule*sec** .

Interpretation

- Slope is same for all targets
- y intercept is different for different target materials.
- $e\Delta V = hf - \phi_{metal}$

□ ϕ is the "work function" of the metal…the mininum amount of energy required to remove an electron.

- $h = 6.63 \times 10^{-34}$ is Planck's Constant!



Interpretation (continued)

- Planck's EM "quanta" turn out to be real after all! (?)
- Light comes in energy packets equal to hf
- Each packet acts more like a particle than a wave
- These light "particles" are called ***photons***
- Rather than continuously absorbing wave radiation the target is being bombarded by photons like tiny billiard balls!

Concluding statements

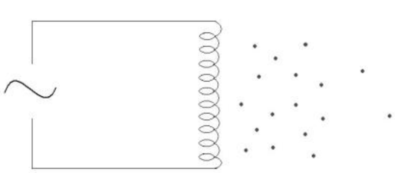
- Einstein figured out the photoelectric effect in 1905 (the same year he developed the theory of special relativity and explained Brownian Motion). This is what he got the Nobel prize for.
- I like to think of the the photoelectric equation in terms of conservation of energy

light energy $\rightarrow$ *ejecting electron & KE*

$KE \rightarrow e\Delta V$

$hf = \phi + KE = \phi + e\Delta V$

Thermionic emission



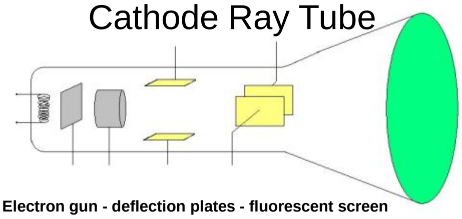
- The emission of electrons from a metal when it is heated

. . . . discovered by Edison

- This paved the way for a revolution in communication

. . . . i.e. radio and television

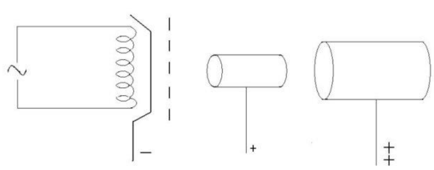
Cathode Ray Tube



Electron gun - deflection plates - fluorescent screen

- Similar to Thomson’s experiment of 1897 where he identified the electron,
- Used in **cathode ray oscilloscopes** for measuring voltages and frequencies.
- Adapted for use in first generation **televisions**

Electron gun



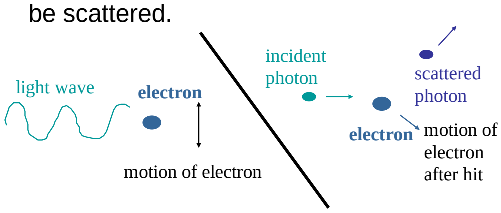
- Accelerating and focussing effect on the electron beam
- **Electron volt** as a unit of energy

Recap questions

- What property of cathode rays suggested that they were charged particles?
- What is meant by the term “ space charge”?
- What is the purpose of the grid in a cathode ray tube?
- How does the grid function?
- How does a cylindrical anode focus the electron beam?
- What happens when cathode rays strike the fluorescent screen
- What is an “electron volt”?

Compton Scattering

- When light encounters charged particles, the particles will interact with the light and cause some of the light to be scattered.



Compton Scattering

From the wave theory, we can understand that charged particles would interact with the light since the light is an electromagnetic wave!

Compton Scattering

- But the actual predictions of how the light scatters from the charged particles **does not fit our simple wave model**.
- If we consider the **photon idea** of light, some of the photons would "hit" the charged particles and "bounce off". The laws of conservation of energy and momentum should then predict the scattering.

Compton Scattering

As we will see in part four of the course, photons DO HAVE MOMENTUM as well as energy.

The scattered photons will have less energy and less momentum after collision with electrons, and so should have a larger wavelength according to the formula:

$\Delta\lambda = \lambda_{scattered} - \lambda_{incident} = (h/mc)[1-\cos(\theta)]$

Compton Scattering

$\Delta\lambda = \lambda_{scattered} - \lambda_{incident} = (h/mc)[1-\cos(\theta)]$

- Note that Planck's constant is in this relation as well, and gives a further experimental way of getting this value.
- Again, the photon theory provides a nice explanation of a phenomenon involving light.

Compton Scattering

$\Delta\lambda = \lambda_{scattered} - \lambda_{incident} = (h/mc)[1-\cos(\theta)]$

Note that the **maximum change in wavelength** is (for scattering from an electron)

$2h/mc = 2(6.63 \times 10^{-34}$ J-s$) / (9.1 \times 10^{-31}$ kg $* 3 \times 10^{8}$ m/s$)$

$=$ **4.86×10^{-12} m $= \Delta\lambda$**

which would be insignificant for visible light (with λ of 10^{-7}m) but NOT for x-ray and γ-ray light (with λ of 10^{-10} m or smaller) .

EXAMPLE

A 17.2-keV x ray from molybdenum is Compton- scattered through an angle of 90°. What is the energy of the x ray after scattering?

Soln: $E = hf \rightarrow \lambda_{incident} = hc\ /\ E\ =\ 0.0721$ nm

$\Delta\lambda = \lambda_{scattered} - \lambda_{incident} = (h/mc)[1\text{-}\cos(\theta)]$

$\lambda_{scattered} = \lambda_{incident} + (h/mc)[1\text{-}\cos(\theta)]$

$\theta\ = 90^o$; $\cos(\theta) = 0$;

Compton wavelength (h/mc) = 0.002426 nm

$\lambda_{scattered} = \lambda_{incident} + (h/mc)[1\text{-}\cos(\theta)] = 0.0721$ nm + 0.0024nm = 0.0745 nm

Energy of the scattered x ray E = hc(eV.nm) / 0.0745 nm =16600eV =16.6keV.

0.6keV missing energy is carried off by the recoil electron

hc = 6.63 e-34 J.s x 3e+17nm/s /1.6e-19 J/eV

WAVE PARTICLE DUALITY

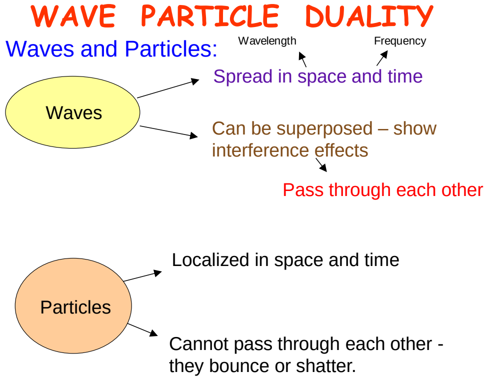
Evidence for wave-particle duality
- Photoelectric effect
- Compton effect

- Electron diffraction
- Interference of matter-waves

Consequence: Heisenberg uncertainty principle

WAVE PARTICLE DUALITY

Waves and Particles:



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WAVE PARTICLE DUALITY

Nature of a wave

- A wave is described by frequency ν, wavelength λ, phase velocity u and intensity I
- A wave is spread out and occupies a relatively large region of space

Nature of a particle

- A particle is specified by mass m, velocity v, momentum p, and energy E
- A particle occupies a definite position in space. In order for that it must be small

WAVE PARTICLE DUALITY

How are they related?

$$\boxed{E = h\nu}$$

- **E**– energy of the photon
- ν– frequency of the wave
- **h**– plank's constant

$$\boxed{p=h/\lambda}$$

- **p** – momentum of the particle
- λ - wavelength of the photon

DE BROGLIE HYPOTHESIS

In the Year 1924 Louis de Broglie made the bold suggestion

LOUIS DE BROGLIE

" If radiation which is basically a wave can exhibit particle nature under certain circumstances, and since nature likes symmetry, then entities which exhibit particle nature ordinarily, should also exhibit wave nature under suitable circumstances"

The reasoning used might be paraphrased as follows

1. **Nature loves symmetry**
2. **Therefore the two great entities, matter and energy, must be mutually symmetrical**
3. **If energy (radiant) is undulatory and/or corpuscular, matter must be corpuscular and/or undulatory**

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WAVE-PARTICLE DUALITY OF LIGHT

In 1924 Einstein wrote:- " There are therefore now two theories of light, both indispensable, and … without any logical connection."

Evidence for wave-nature of light
- Diffraction and interference

Evidence for particle-nature of light
- Photoelectric effect
- Compton effect

- Light exhibits diffraction and interference phenomena that are *only* explicable in terms of wave properties
- Light is always detected as packets (photons); if we look, we never observe half a photon
- Number of photons proportional to energy density (i.e. to square of electromagnetic field strength)

The de Broglie Hypothesis

- If light can act like a wave sometimes and like a particle at other times, then *all matter*, usually thought of as particles, should exhibit wave-like behaviour
- The relation between the momentum and the wavelength of a photon can be applied to material particles also

Prince Louis de Broglie (1892-1987)

de Broglie Wavelength

wavelength

$$\lambda = \frac{h}{p}$$

momentum

Relates a particle-like property (p) to a wave-like property (λ)

DE BROGLIE WAVELENGTH

The Wave associated with the matter particle is called Matter Wave.

The Wavelength associated is called de Broglie Wavelength.

$$\text{de Broglie wavelength } \lambda = \frac{h}{p} = \frac{h}{mv}$$

h is Planck's Constant
m is the mass of the particle
v is the velocity of the particle

for an electron with Kinetic Energy 'E'
accelerated by a Potential difference 'V'

$$\text{Then} \quad \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

substituting for h, m, and e we get

$$\lambda = \frac{6.625\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.602\times10^{-19}\times V}} = \frac{1.226}{\sqrt{V}}\ nm$$

thus for V = 100 Volts

$$\lambda = \frac{1.226}{\sqrt{100}} = 0.1226\ nm$$

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WAVE PARTICLE DUALITY

Evidence for wave-particle duality
- Photoelectric effect
- Compton effect

Wave theory

- Electron diffraction
- Interference of matter-waves

Particle theory

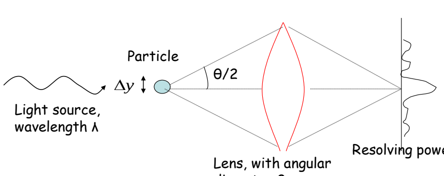
Consequence: Heisenberg uncertainty principle

HEISENBERG UNCERTAINTY PRINCIPLE

(also called the Bohr microscope, but the thought experiment is mainly due to Heisenberg).

The microscope is an imaginary device to measure the position (y) and momentum (p) of a particle.

Heisenberg



Resolving power of lens:

$$\Delta y \geq \frac{\lambda}{\theta}$$

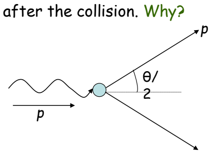
HEISENBERG (cont)

Photons transfer momentum to the particle when they scatter.

Magnitude of p is the same before and after the collision. Why?

Uncertainty in *photon* y-momentum
= Uncertainty in *particle* y-momentum

$$\boxed{-p\sin(\theta/2) \le p_y \le p\sin(\theta/2)}$$



Small angle approximation

$$\Delta p_y = 2p\sin(\theta/2) \approx p\theta$$

de Broglie relation gives $p = h/\lambda$ and so $\Delta p_y \approx \dfrac{h\theta}{\lambda}$

From before $\Delta y \ge \dfrac{\lambda}{\theta}$ hence $\boxed{\Delta p_y \Delta y \approx h}$

HEISENBERG UNCERTAINTY PRINCIPLE.

HEISENBERG UNCERTAINTY PRINCIPLE

$$\boxed{\begin{aligned} \Delta x \Delta p_x &\ge \hbar/2 \\ \Delta y \Delta p_y &\ge \hbar/2 \\ \Delta z \Delta p_z &\ge \hbar/2 \end{aligned}}$$

HEISENBERG UNCERTAINTY PRINCIPLE.

We cannot have simultaneous knowledge
of 'conjugate' variables such as position and momenta.

Note, however, $\boxed{\Delta x \Delta p_y \ge 0}$ etc

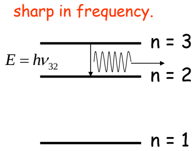
Arbitrary precision is possible in principle for
position in one direction and momentum in another

HEISENBERG UNCERTAINTY PRINCIPLE

There is also an energy-time uncertainty relation

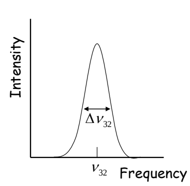
$$\boxed{\Delta E \Delta t \ge \hbar/2}$$

Transitions between energy levels of atoms are not perfectly sharp in frequency.



An electron in n = 3 will spontaneously decay to a lower level after a lifetime of order $\sim 10^{-8}$ s

There is a corresponding 'spread' in the emitted frequency



SUMMARY OF PHOTON PROPERTIES

Relation between particle and wave properties of light

Energy and frequency $E = h\nu$

Also have relation between momentum and wavelength

Relativistic formula relating energy and momentum $\Rightarrow$ $\boxed{E^2 = p^2c^2 + m^2c^4}$

For light $E = pc$ and $c = \lambda\nu$

$$\boxed{p = \frac{h}{\lambda} = \frac{h\nu}{c}}$$

Also commonly write these as

$E = \hbar\omega$ $\quad p = \hbar k$ $\quad \omega = 2\pi\nu$ (angular frequency) $\quad k = \frac{2\pi}{\lambda}$ (wavevector) $\quad \hbar = \frac{h}{2\pi}$ (hbar)

CONCLUSIONS

Light and matter exhibit wave-particle duality

Relation between wave and particle properties given by the de Broglie relations $\boxed{E = h\nu \quad p = \frac{h}{\lambda}}$

Evidence for particle properties of light
Photoelectric effect, Compton scattering

Evidence for wave properties of matter
Electron diffraction, interference of matter waves (electrons, neutrons, He atoms, C60 molecules)
Heisenberg uncertainty principle limits simultaneous knowledge of conjugate variables

$$\boxed{\begin{aligned} \Delta x\Delta p_x &\geq \hbar/2 \\ \Delta y\Delta p_y &\geq \hbar/2 \\ \Delta z\Delta p_z &\geq \hbar/2 \end{aligned}}$$