Lenses and Optical Instruments

Units

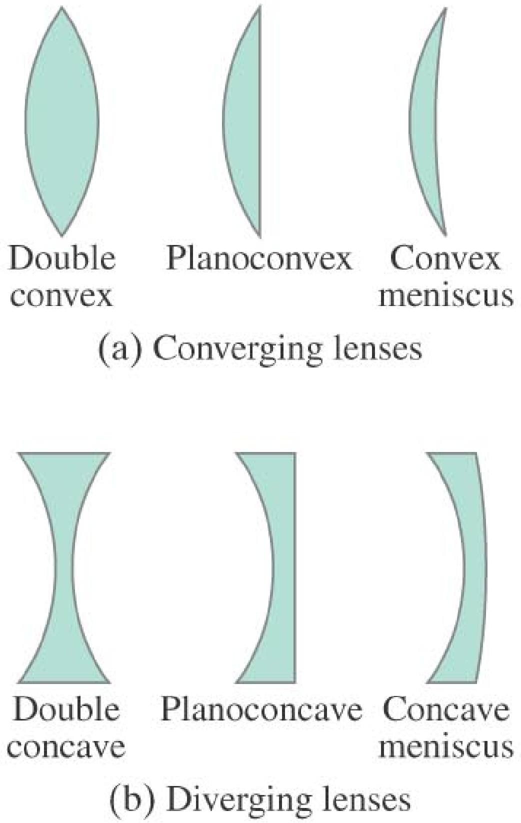
- Thin Lenses; Ray Tracing
- The Thin Lens Equation; Magnification
- Combinations of Lenses
- Lensmaker's Equation
- Cameras: Film and Digital
- The Human Eye; Corrective Lenses
- Magnifying Glass
- Telescopes
- Compound Microscope
- Aberrations of Lenses and Mirrors

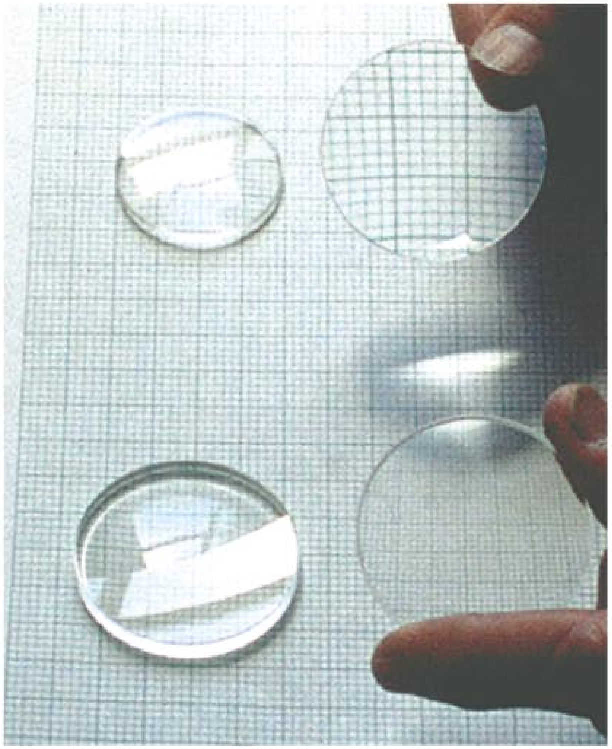
Units of Chapter 33

- Telescopes
- Compound Microscope
- Aberrations of Lenses and Mirrors

33–1 Thin Lenses; Ray Tracing

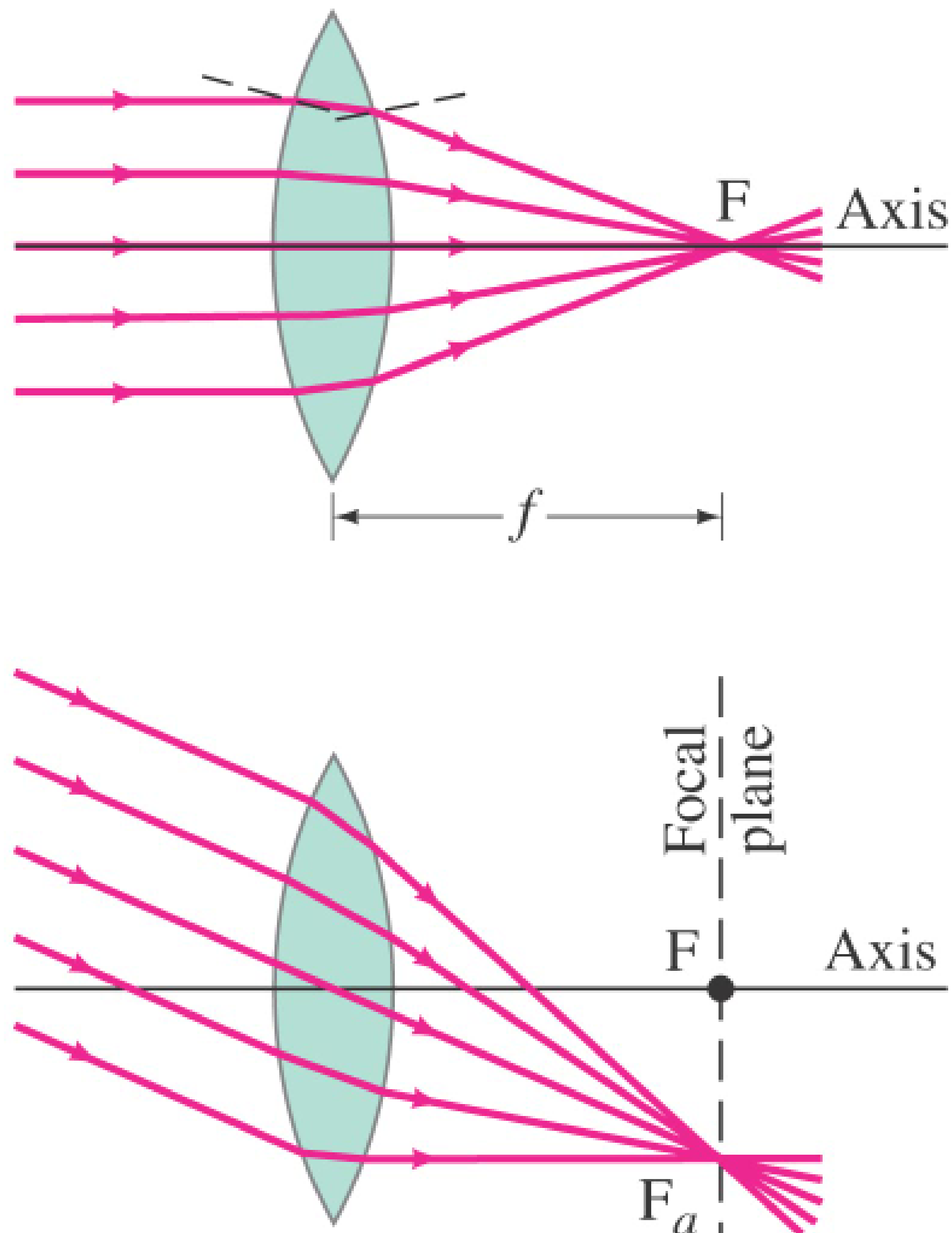
Thin lenses are those whose thickness is small compared to their radius of curvature. They may be either converging (a) or diverging (b).





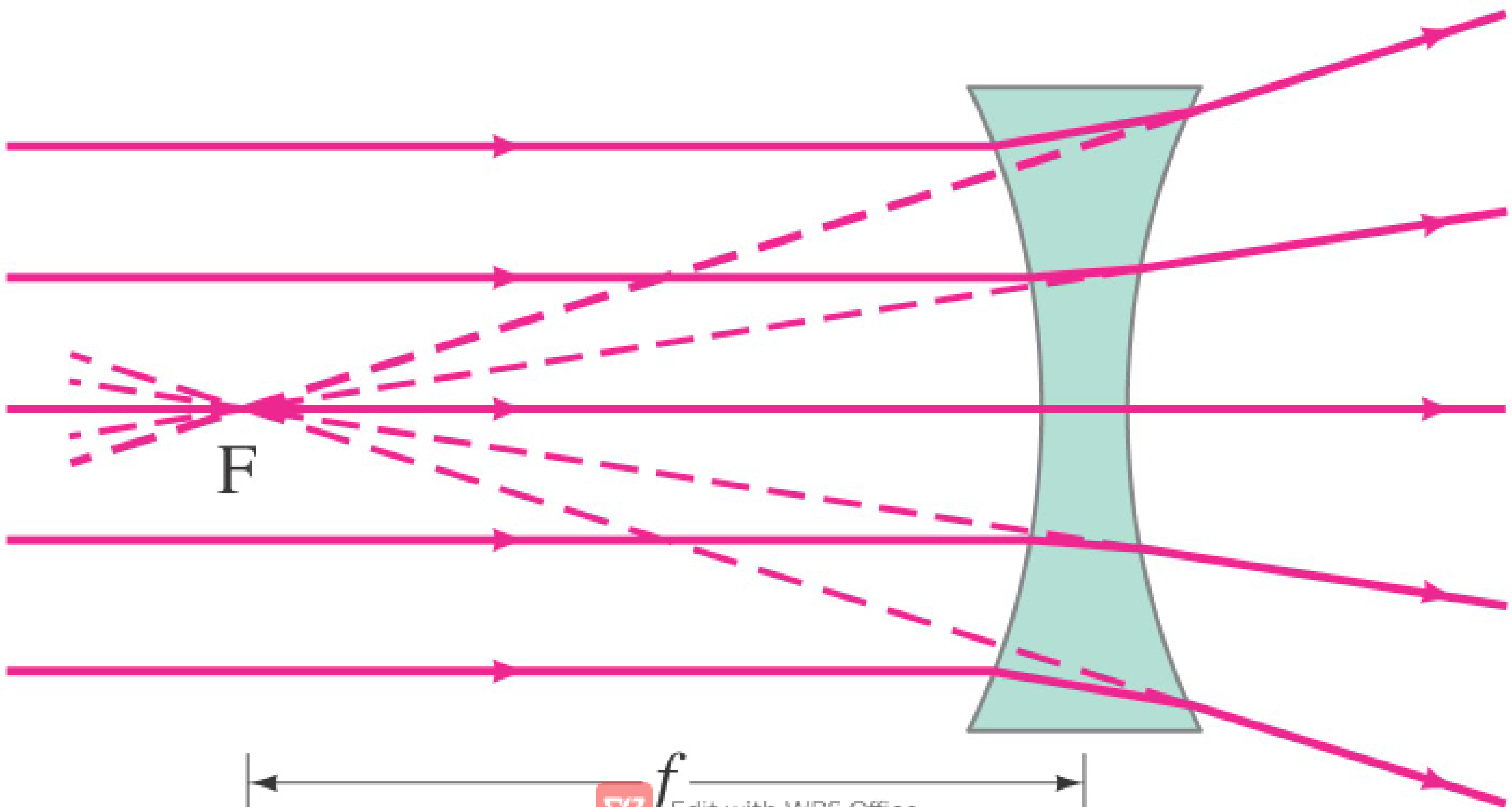
33–1 Thin Lenses; Ray Tracing

Parallel rays are brought to a focus by a converging lens (one that is thicker in the center than it is at the edge).



33-1 Thin Lenses; Ray Tracing

A diverging lens (thicker at the edge than in the center) makes parallel light diverge; the focal point is that point where the diverging rays would converge if projected back.



33–1 Thin Lenses; Ray Tracing

The power of a lens is the inverse of its focal length:

$$P = \frac{1}{f}.$$

Lens power is measured in diopters, D:

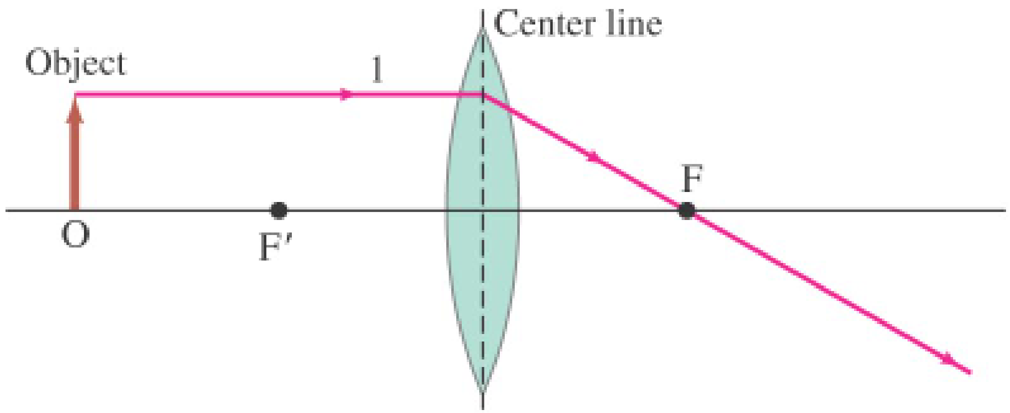
$$1\ \mathrm{D} = 1\ \mathrm{m}^{-1}.$$

33-1 Thin Lenses; Ray Tracing

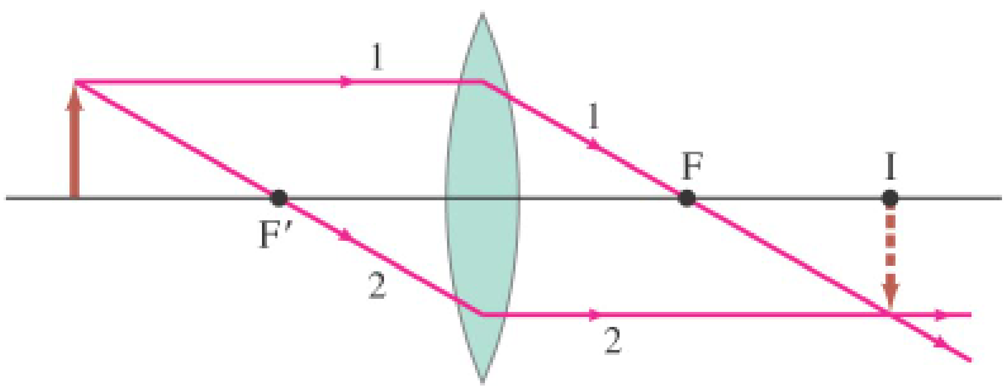
Ray tracing for thin lenses is similar to that for mirrors. We have three key rays:

1. This ray comes in parallel to the axis and exits through the focal point.
2. This ray comes in through the focal point and exits parallel to the axis.
3. This ray goes through the center of the lens and is undeflected.

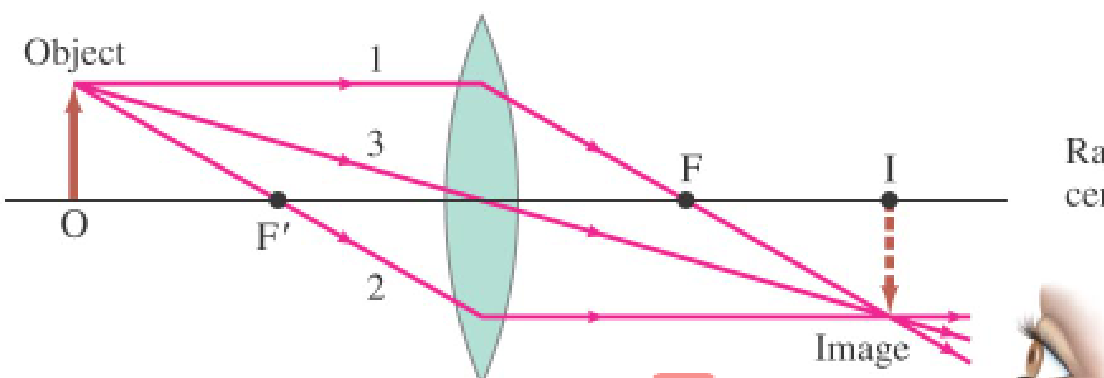
33–1 Thin Lenses; Ray Tracing



Ray 1 leaves one point on object going parallel to the axis, then refracts through focal point behind the lens.



Ray 2 passes through F′ in front of the lens; therefore it is parallel to the axis behind the lens.



Ray 3 passes straight through the center of the lens (assumed very thin).

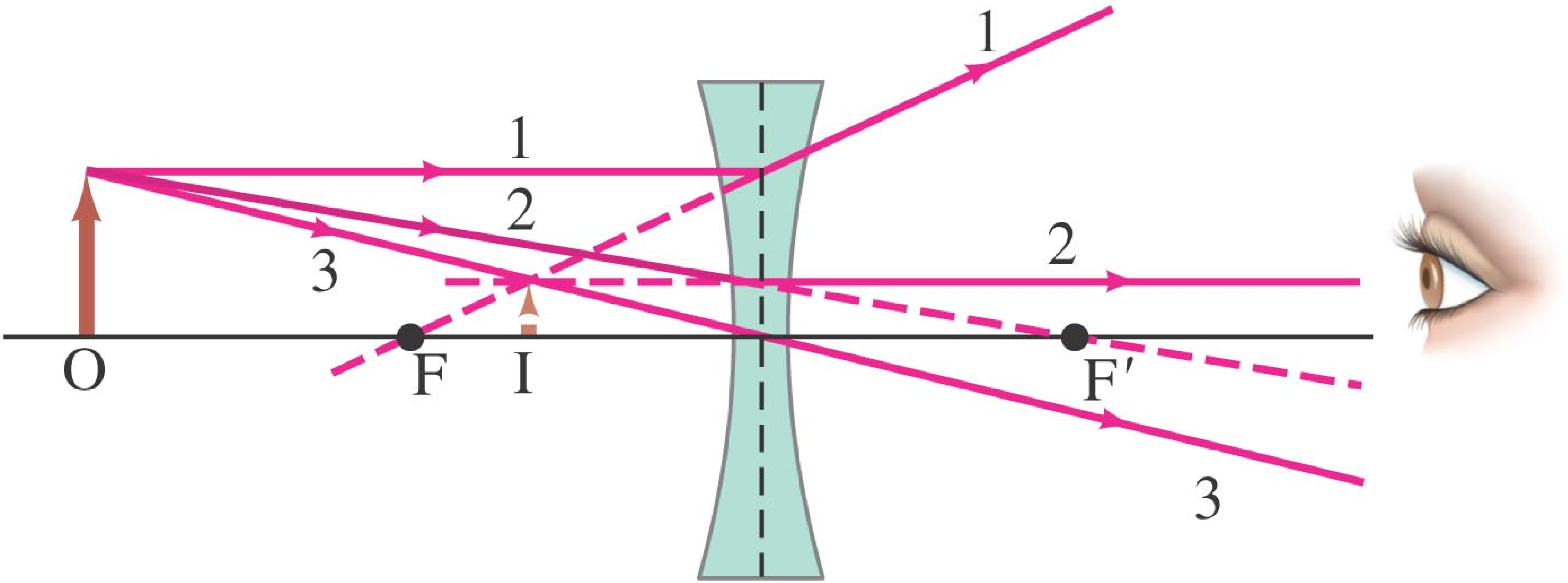
33-1 Thin Lenses; Ray Tracing

Conceptual Example 33-1: Half-blocked lens.

What happens to the image of an object if the top half of a lens is covered by a piece of cardboard?

33–1 Thin Lenses; Ray Tracing

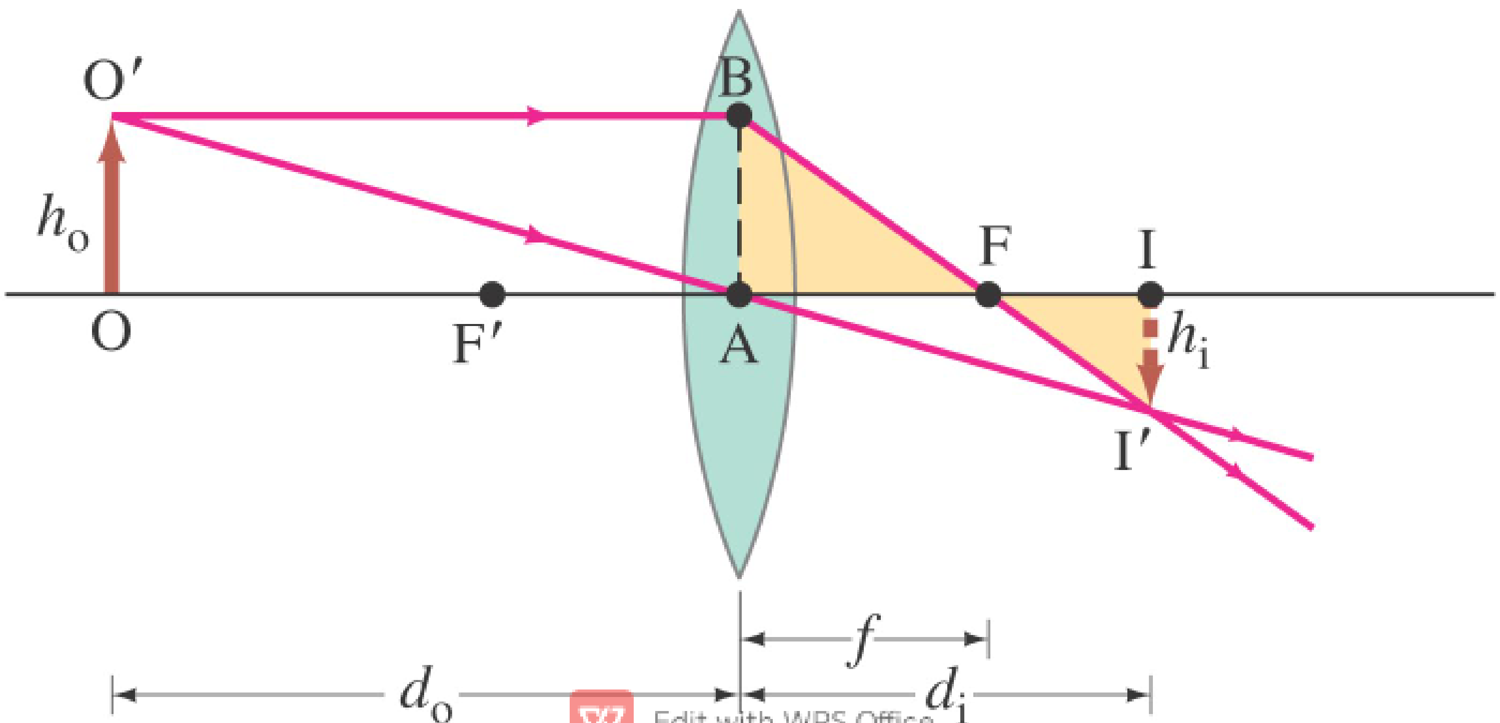
For a diverging lens, we can use the same three rays; the image is upright and virtual.



33-2 The Thin Lens Equation; Magnification

The thin lens equation is similar to the mirror equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.$$



33-2 The Thin Lens Equation; Magnification

The sign conventions are slightly different:

1. The focal length is positive for converging lenses and negative for diverging.
2. The object distance is positive when the object is on the same side as the light entering the lens (not an issue except in compound systems); otherwise it is negative.
3. The image distance is positive if the image is on the opposite side from the light entering the lens; otherwise it is negative.
4. The height of the image is positive if the image is upright and negative otherwise.

33-2 The Thin Lens Equation; Magnification

The magnification formula is also the same as that for a mirror:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}.$$

The power of a lens is positive if it is converging and negative if it is diverging.

33-2 The Thin Lens Equation; Magnification

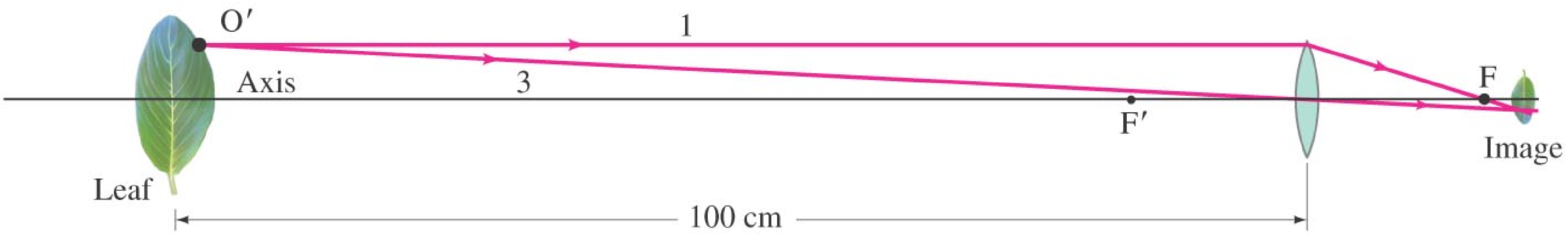
Problem Solving: Thin Lenses

1. Draw a ray diagram. The image is located where the key rays intersect.
2. Solve for unknowns.
3. Follow the sign conventions.
4. Check that your answers are consistent with the ray diagram.

33-2 The Thin Lens Equation; Magnification

Example 33-2: Image formed by converging lens.

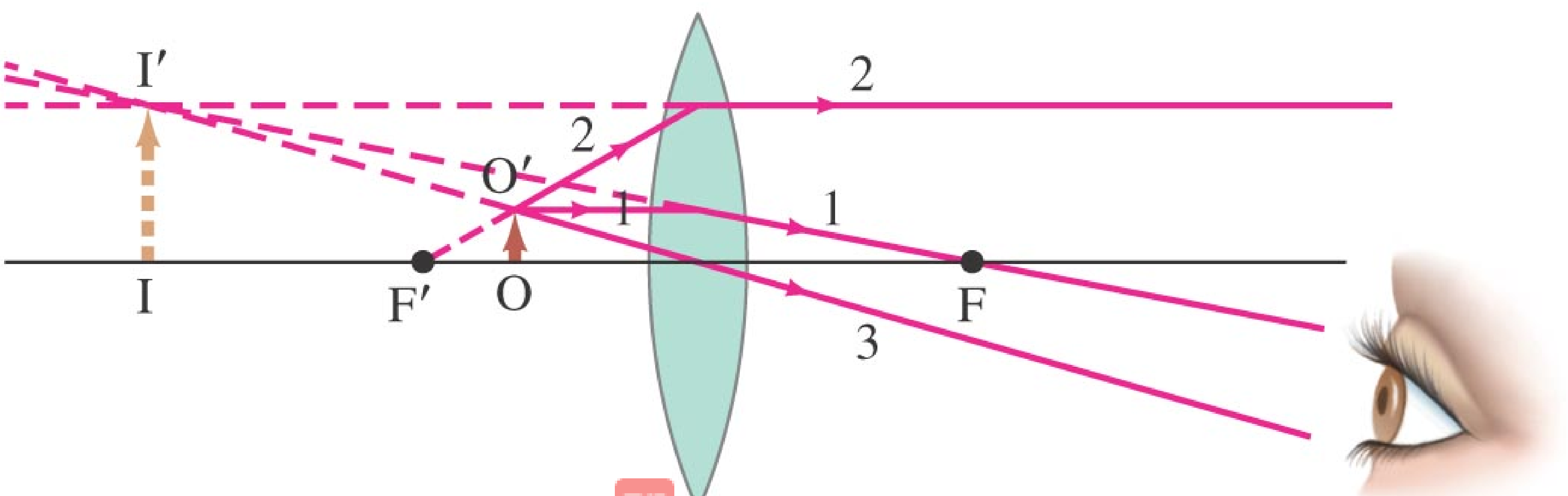
What are (a) the position, and (b) the size, of the image of a 7.6-cm-high leaf placed 1.00 m from a +50.0-mm-focal-length camera lens?



33-2 The Thin Lens Equation; Magnification

Example 33-3: Object close to converging lens.

An object is placed 10 cm from a 15-cm-focal-length converging lens. Determine the image position and size (a) analytically, and (b) using a ray diagram.



33-2 The Thin Lens Equation; Magnification

Example 33-4: Diverging lens.

Where must a small insect be placed if a 25-cm-focal-length diverging lens is to form a virtual image 20 cm from the lens, on the same side as the object?

33-3 Combinations of Lenses

Optical instruments use lenses in combination. When light passes through more than one lens, we find the image formed by the first lens as if it were alone. This image becomes the object for the second lens (this is where object distances may be negative), and we find the image then formed by this second lens, which is the final image if there are only two lenses

The total magnification is the product of the magnification of each lens.

33-4 Combinations of Lenses

Add together the powers of each of the two lenses to get the power of the combined lens.

Ex:

Lens A has $f_A = + 0.5$ m

Lens B has $f_B = -1.0$ m

What is the power of each lens? ; $P_A = 2D$; $P_B = -1D$

What is the power of the combined lens when the two lenses are held together? ; $P_{co} = 2-1 = 1D$

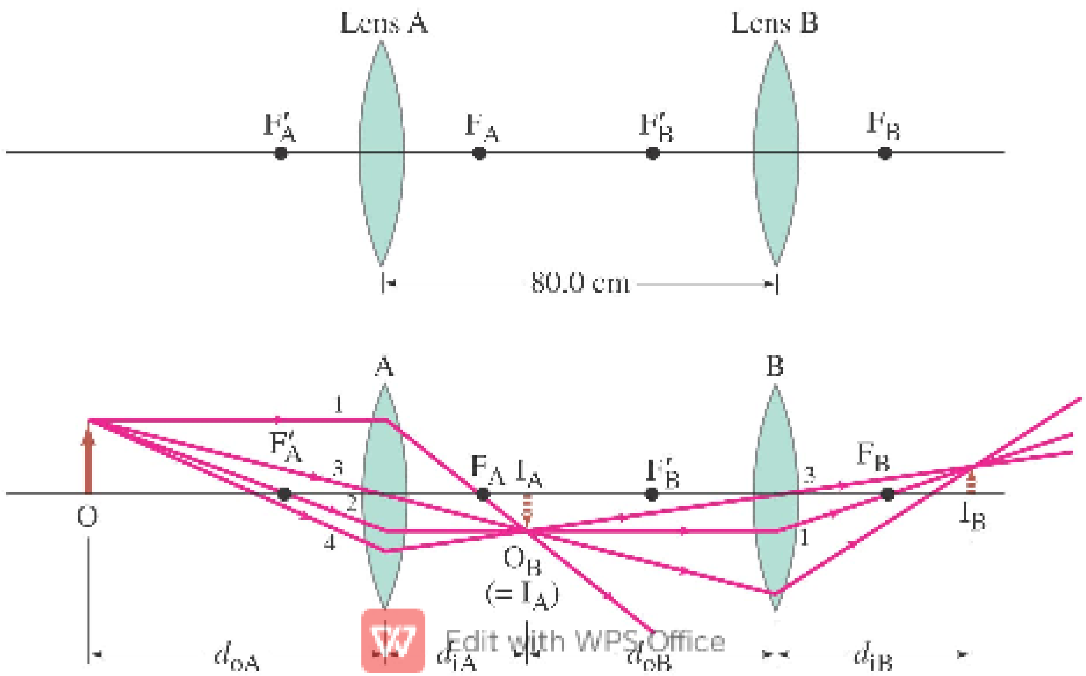
What is the focal length of the combined lens? $F_{co} = 1$ m

Note: This is only for THIN LENSES held in contact

33-3 Combinations of Lenses

Example 33-5: A two-lens system.

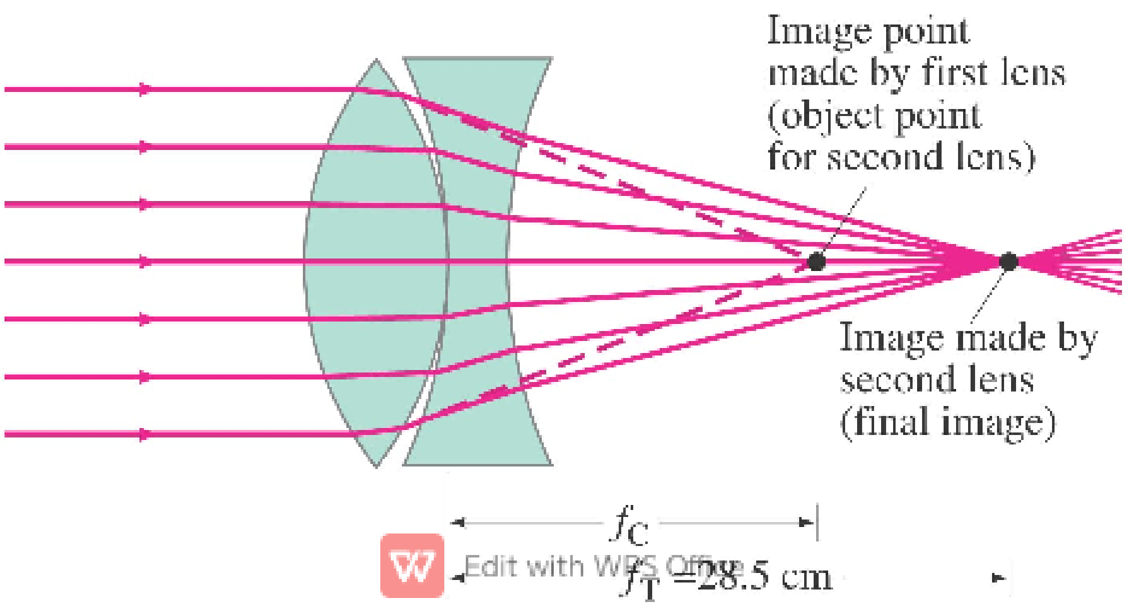
Two converging lenses, A and B, with focal lengths $f_A = 20.0$ cm and $f_B = 25.0$ cm, are placed 80.0 cm apart. An object is placed 60.0 cm in front of the first lens. Determine (a) the position, and (b) the magnification, of the final image formed by the combination of the two lenses.



33-3 Combinations of Lenses

Example 33-6: Measuring *f* for a diverging lens.

To measure the focal length of a diverging lens, a converging lens is placed in contact with it. The Sun's rays are focused by this combination at a point 28.5 cm behind the lenses as shown. If the converging lens has a focal length f_C of 16.0 cm, what is the focal length f_D of the diverging lens? Assume both lenses are thin and the space between them is negligible.



33–3 Combinations of Lenses

Solution: Measuring *f* for a diverging lens.

The image distance for the first lens equals its focal length (16.0cm) since the object distance is at infinity

The position of this image, even though it is never actually formed, acts as the object for the second (diverging lens).

Applying thin lens equation to the concave lens

$1/f_d = 1/d_o + 1/d_i = -1/16 + 1/28.5 = -0.027$

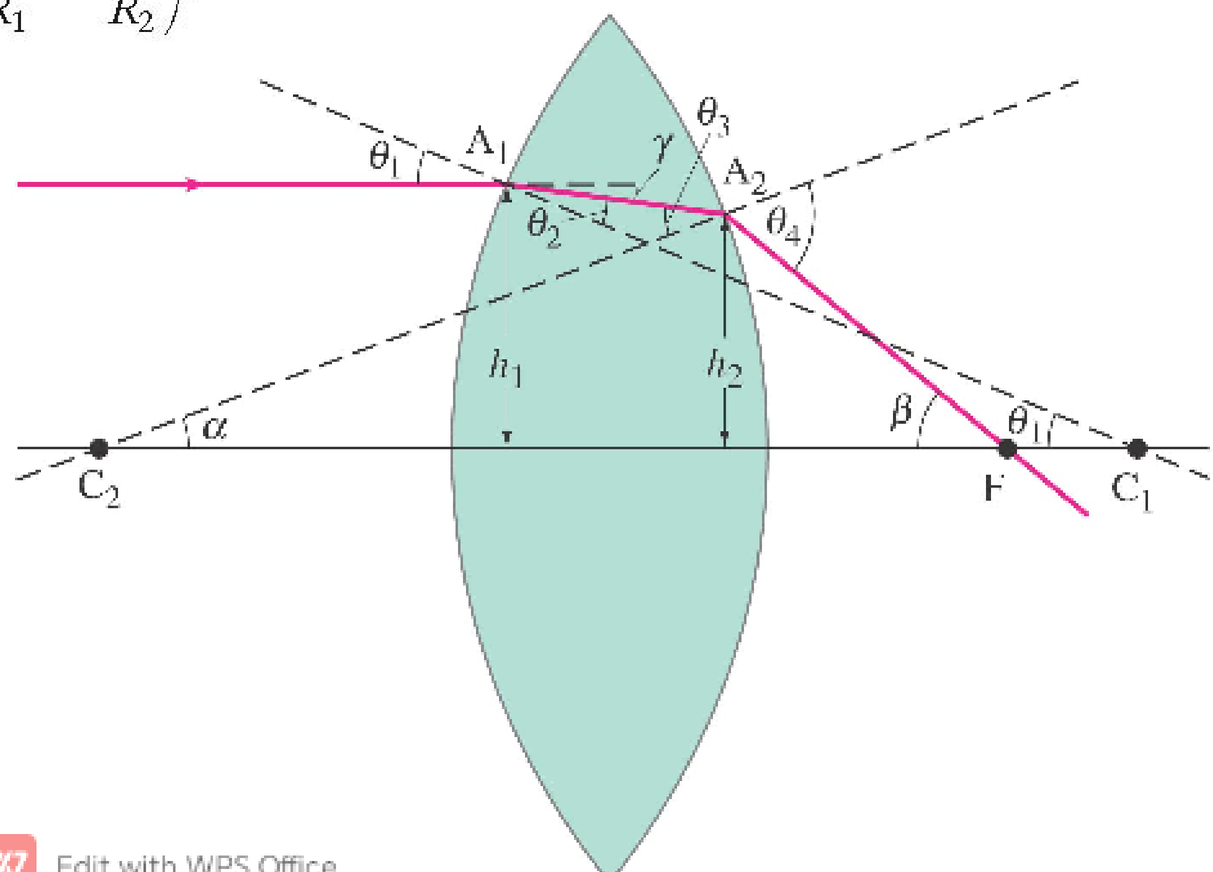
$f_d = -36.5$ cm

33–4 Lensmaker's Equation

This useful equation relates the radii of curvature of the two lens surfaces, and the index of refraction, to the focal length:

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

Note that f for a thin lens does not depend on h_1 or h_2. The position of the point F does not depend on where the ray strikes the lens.



Lensmaker's Equation cont'd

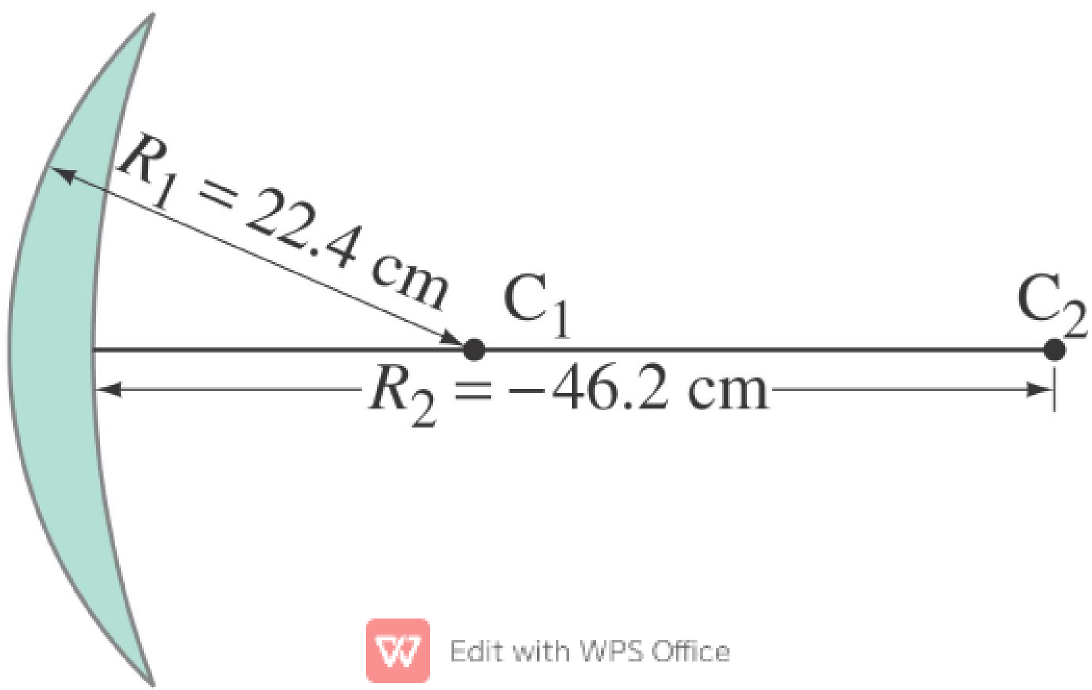
In the equation above, both surfaces are convex and R_1 and R_2 are considered positive. It also works for lenses with one or both surfaces concave; but for concave surface, the radius must be considered negative.

The equation is symmetrical in R_1 and R_2. Thus, if a lens is turned around so that light impinges on the other surface, the focal length is same even if the two lens surfaces are different.

33-4 Lensmaker's Equation

Example 33-7: Calculating *f* for a converging lens.

A convex meniscus lens is made from glass with $n = 1.50$. The radius of curvature of the convex surface is 22.4 cm and that of the concave surface is 46.2 cm. (a) What is the focal length? (b) Where will the image be for an object 2.00 m away?



(a) R_1 = 22.4 cm and R_2 = - 46.2cm

Using $$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

$1/f = (1.50 – 1.00) (1/22.4 – 1/ 46.2)$

$f = 87$ cm

As such the lens is converging. What happens if we turn the lens around?

(b) From the lens equation, with f = 0.87m and d_o = 2.0 m, we have

$1/d_i = 1/f – 1/d_o = 0.649$ m

d_i = 1.54 m

Ex.

A converging glass lens with index of refraction of 1.52 has a focal length 14 cm in air. What is its focal length when it is immersed in water, which has an index of refraction of 1.33 ?

$$\frac{1}{fair} = (n-1)\left(\frac{1}{r1} - \frac{1}{r2}\right)$$

$$\frac{1}{fwater} = (n'-1)\left(\frac{1}{r1} - \frac{1}{r2}\right)$$

$$n' = ratiooftheindexofrefractionofglasstowater$$

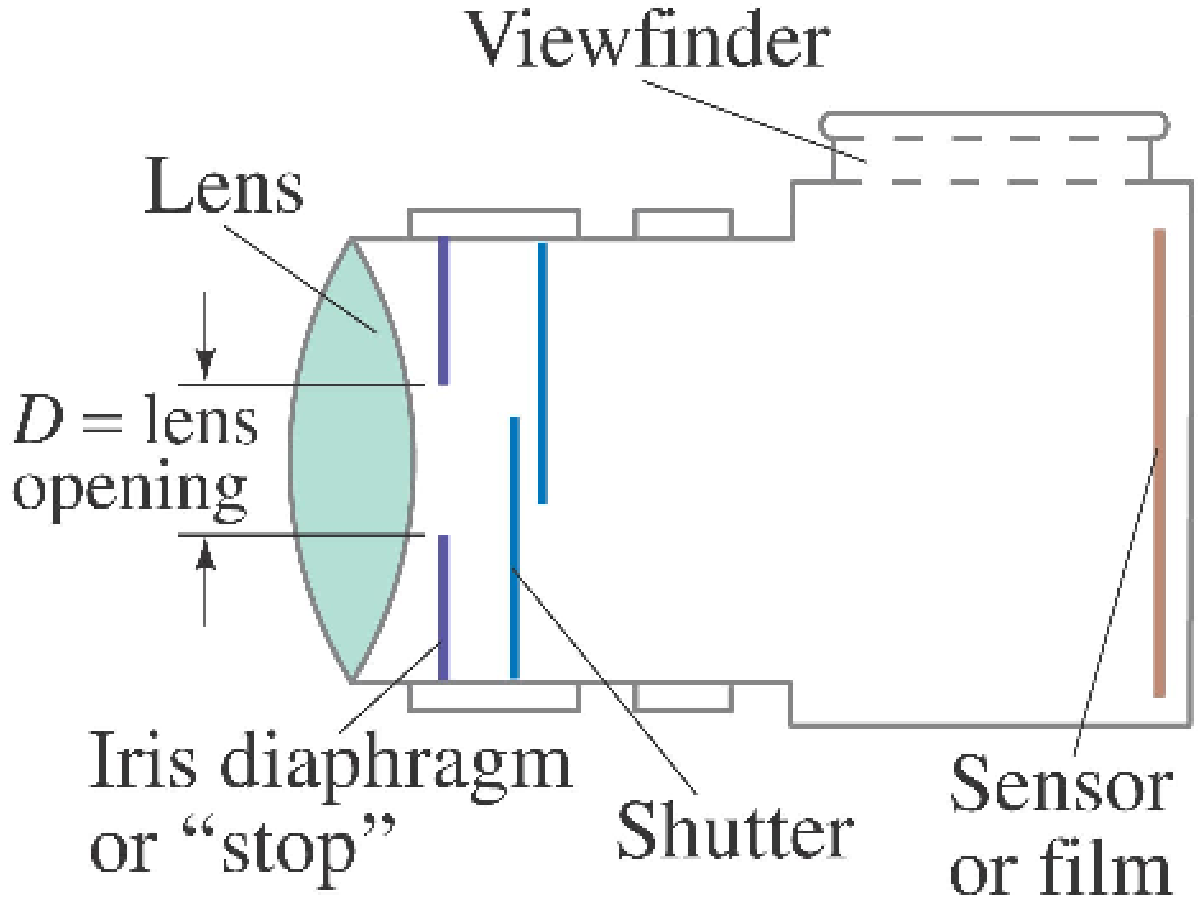
$$n' = 1.52/1.33 = 1.14$$

$$\frac{fwater}{fair} = \frac{n-1}{n'-1} = \frac{1.52-1}{1.14-1} = 3.71$$

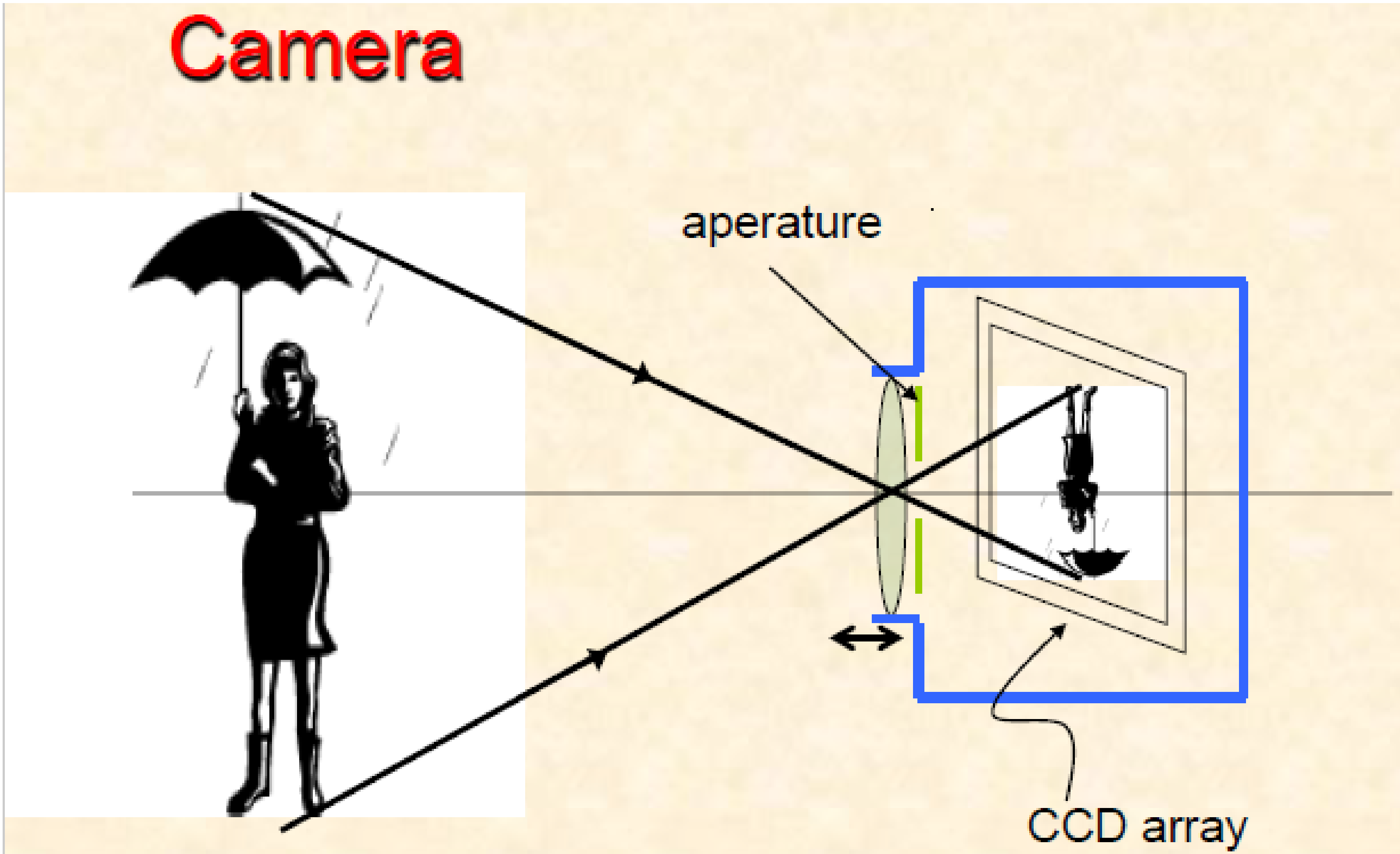
33–5 Cameras: Film and Digital

Basic parts of a camera:

- Lens
- Light-tight box
- Shutter
- Film or electronic sensor



Camera cont'd

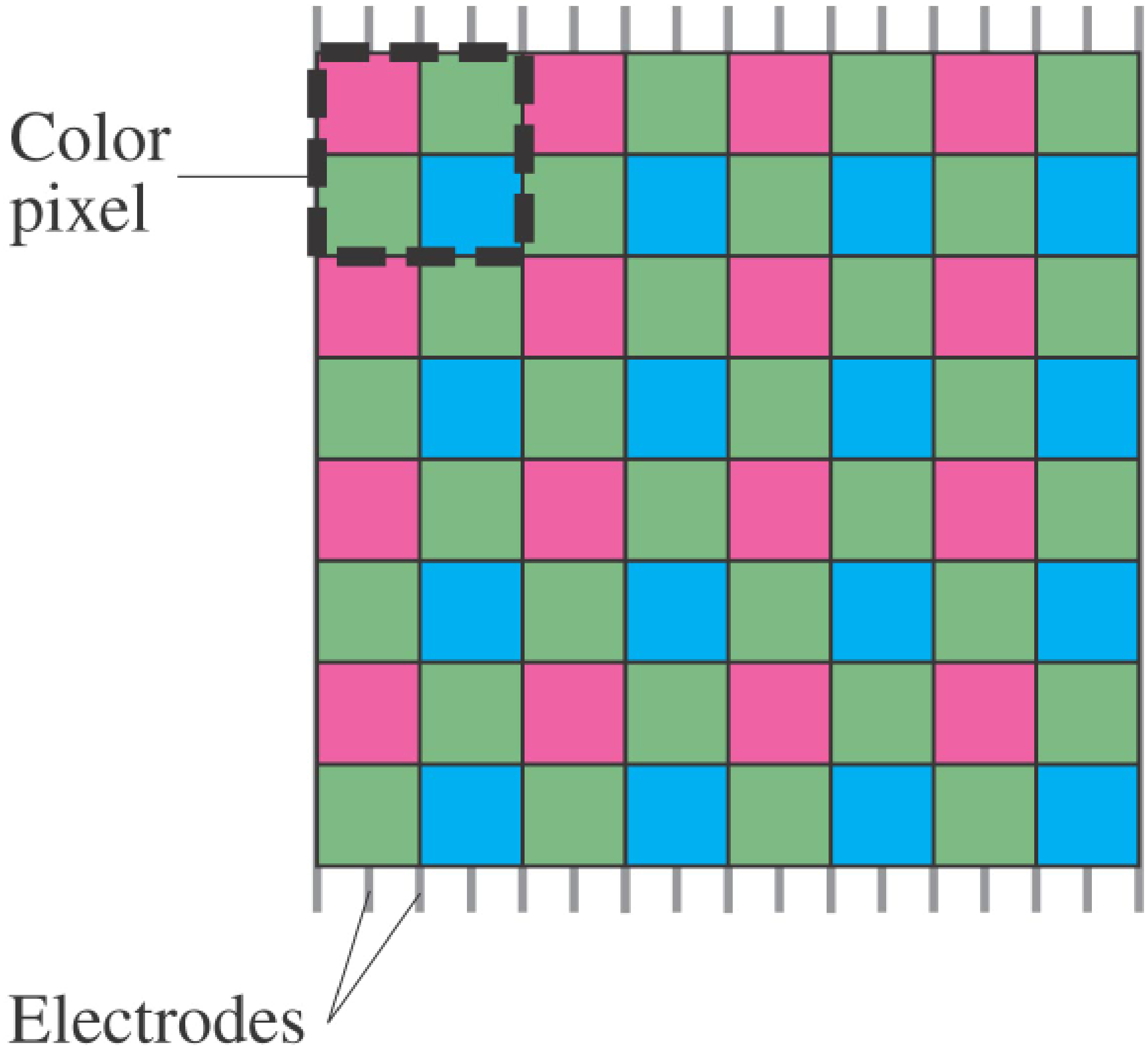


Lens translated to change image distance S' to adjust for different object distances S.

Focal length of lens is fixed.

33–5 Cameras: Film and Digital

A digital camera uses CCD sensors instead of film. The digitized image is sent to a processor for storage and later retrieval.



33–5 Cameras: Film and Digital

Camera adjustments:

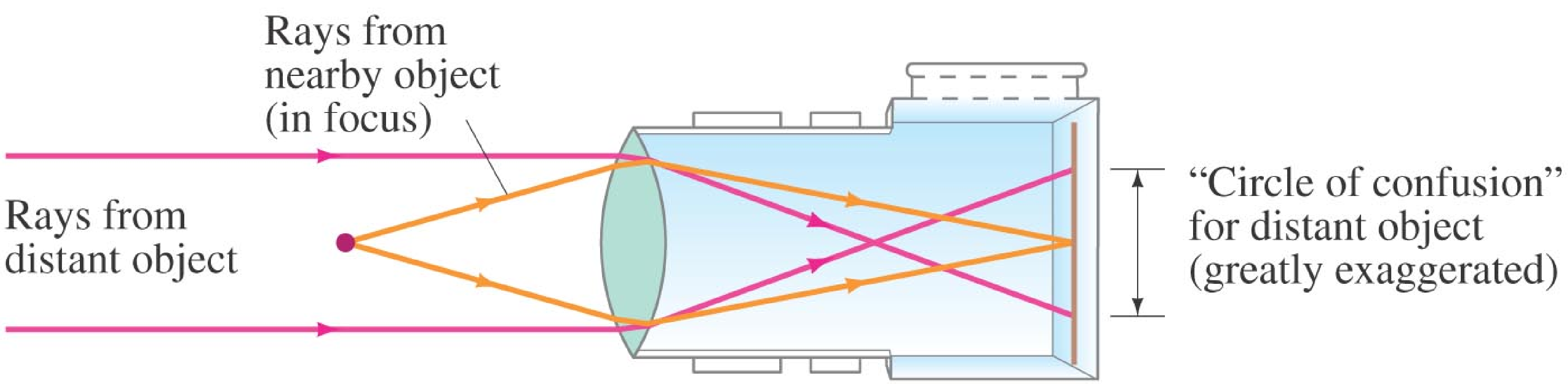
• Shutter speed: controls the amount of time light enters the camera. A faster shutter speed makes a sharper picture.

• *f*-stop: controls the maximum opening of the shutter. This allows the right amount of light to enter to properly expose the film, and must be adjusted for external light conditions.

• Focusing: this adjusts the position of the lens so that the image is positioned on the film.

33–5 Cameras: Film and Digital

There is a certain range of distances over which objects will be in focus; this is called the depth of field of the lens. Objects closer or farther will be blurred.



33–5 Cameras: Film and Digital

Example 33–8: How far must a 50.0-mm-focal-length camera lens be moved from its infinity setting to sharply focus an object 3.00 m away?

Solution: For an object at infinity, the image is at the focal point.

Object distance u = 3m , using thin lens equation, we can find the image distance (distance of lens to sensor)

When focused at infinity, the lens is 50 mm from the film. When focused at 3 m, the image distance v is 50.8 cm. So the lens must move 0.8 mm away from sensor

33–5 Cameras: Film and Digital

Conceptual Example 33–9: Shutter speed.

To improve the depth of field, you "stop down" your camera lens by two *f*-stops from *f*/4 to *f*/8. What should you do to the shutter speed to maintain the same exposure?

33-5 Cameras: Film and Digital

Example 33-10: Pixels and resolution.

A 6-MP (6-megapixel) digital camera offers a maximum resolution of 2000 x 3000 pixels on a 16-mm x 24-mm CCD sensor. How sharp should the lens be to make use of this resolution?

33-5 Cameras: Film and Digital

Example 33-11: Blown-up photograph.

An enlarged photograph looks sharp at normal viewing distances if the dots or lines are resolved to about 10 dots/mm. Would an 8 x 10-inch enlargement of a photo taken by the camera in Example 33-10 seem sharp? To what maximum size could you enlarge this 2000 x 3000-pixel image?

33-5 Cameras: Film and Digital

There are different types of lenses available for cameras, besides the normal lens.

Telephoto lens: longer focal length, magnified image

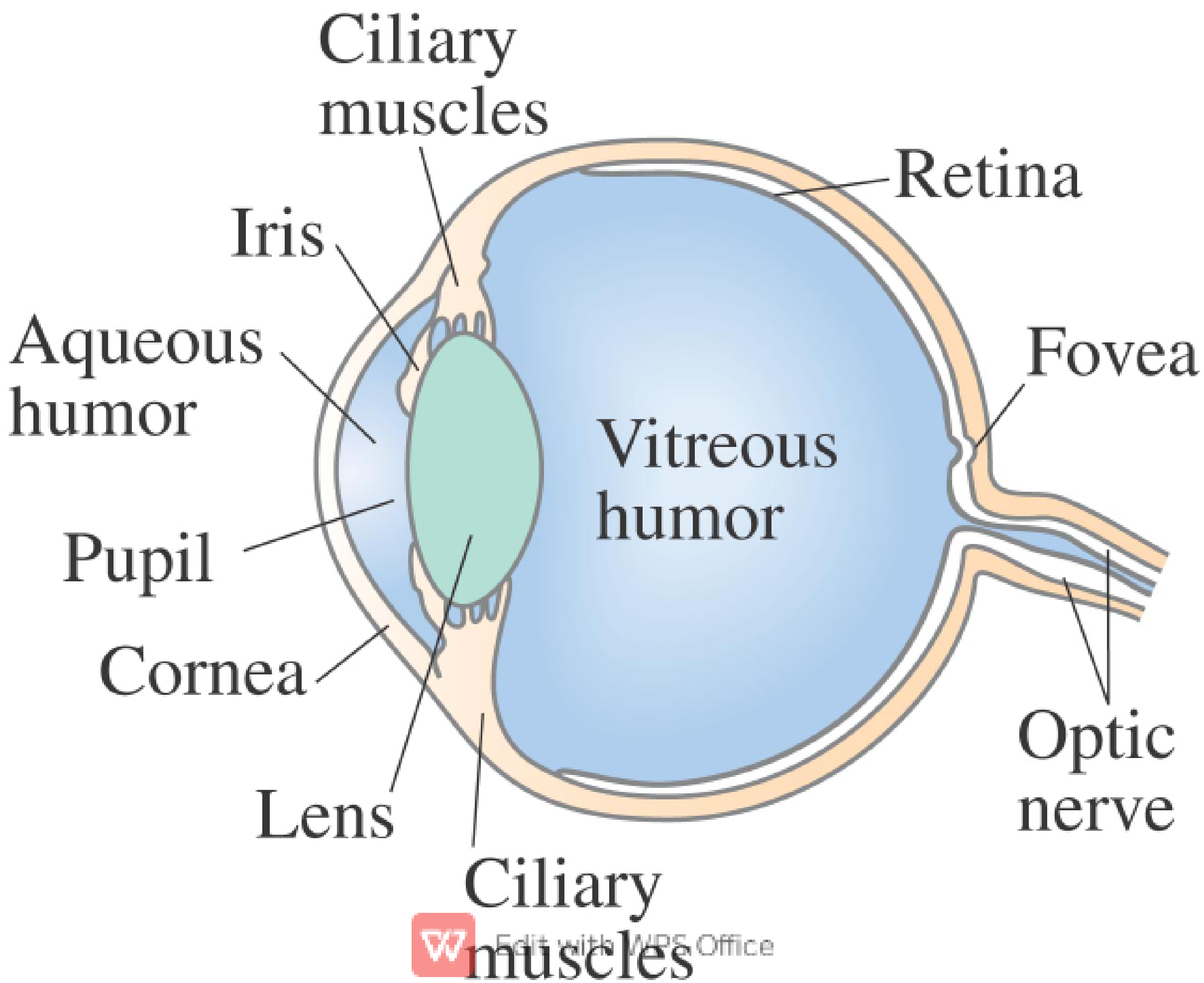
Wide-angle lens: shorter focal length, wider field of view, smaller image

Zoom lens: adjustable focal length

Digital zoom (in digital cameras): enlarges pixels with loss of resolution

33-6 The Human Eye; Corrective Lenses

The human eye resembles a camera in its basic functioning, with an adjustable lens, the iris, and the retina.

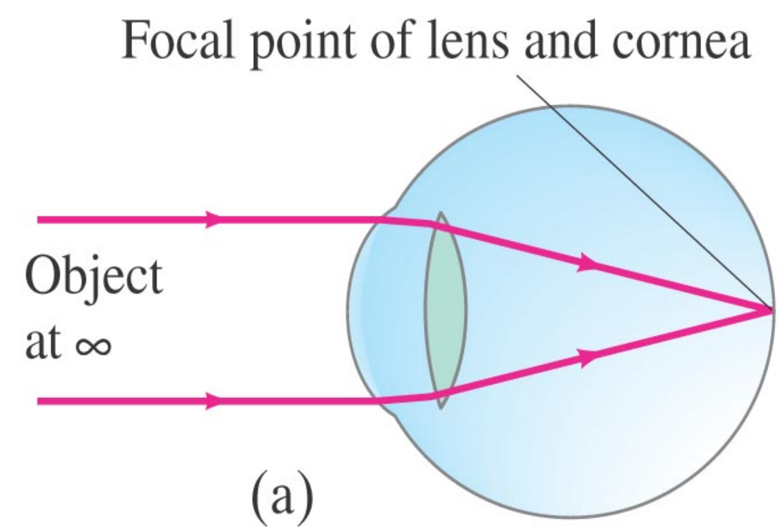


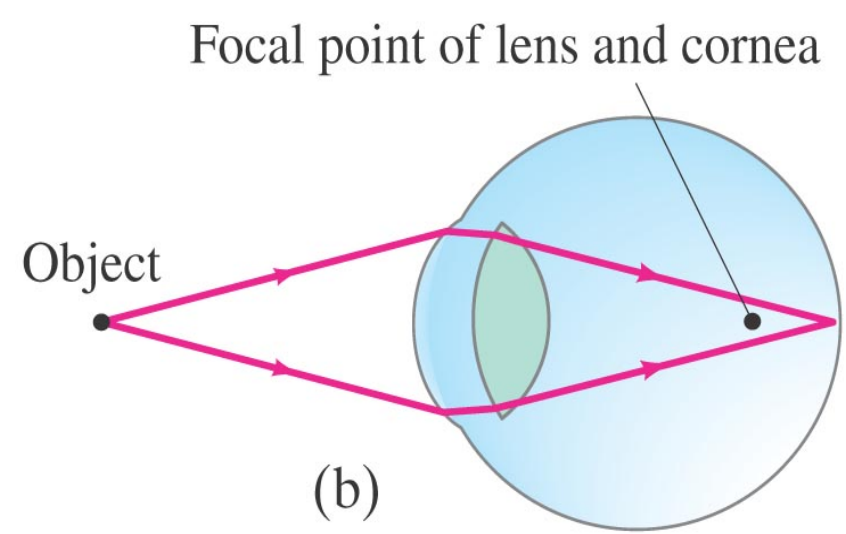
33-6 The Human Eye; Corrective Lenses

The lens of the eye (n = 1.386 to 1.406) does little of the bending of the light rays.

The lens acts as a fine adjustment for focusing at different distances

Most of the refraction is done at the surface of the cornea; the lens makes small adjustments to focus at different distances.





33-6 The Human Eye; Corrective Lenses

To focus on a nearby object, the muscles contract, causing the centre of the lens to thicken, thus shortening the focal length so that images of nearby objects can be focused on the retina. This focusing adjustment is called accommodation

Near point: closest distance at which eye can focus clearly. Normal is about 25 cm.

Far point: farthest distance at which object can be seen clearly. Normal is at infinity.

Nearsightedness: far point is too close.

Farsightedness: near point is too far away.

33-6 The Human Eye; Short-sight

Short-sighted people can focus on nearby objects, but cannot focus on distant ones .

The far point is nearer to the eye than infinity. They are said to have myopia.

Someone suffering from myopia has cornea and lens combination that is too powerful, or an eyeball that is too long.

Distant objects are brought to a focus before the light reaches the retina.

A diverging lens is needed to correct this.

33-6 The Human Eye; Long-sight

When the lens- cornea combination is not powerful enough, the light from nearby objects cannot be brought to a focus on the retina.

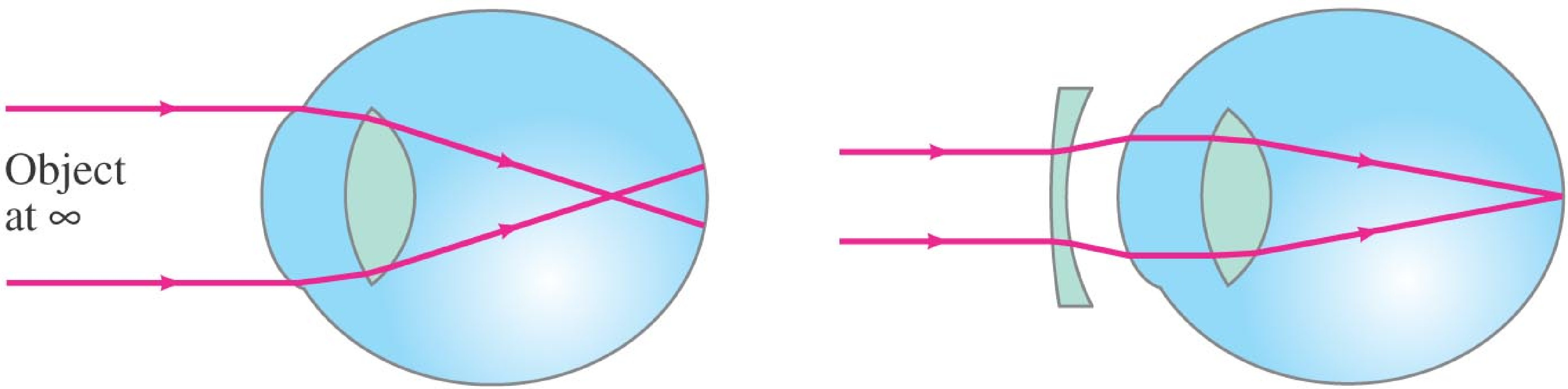
The near point is further away than the normal 25cm. This is long sight, or hypermetropia.

Someone suffering from this defect can focus on objects at infinity, but not on nearby things

A converging lens is needed to correct this as it helps to converge the light more.

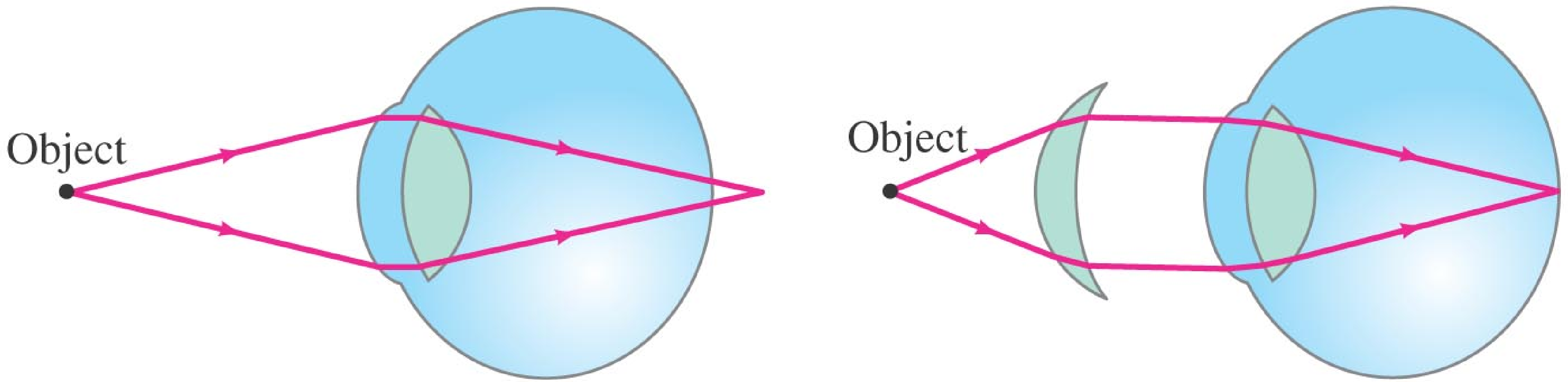
33-6 The Human Eye; Corrective Lenses

Nearsightedness can be corrected with a diverging lens.



33-6 The Human Eye; Corrective Lenses

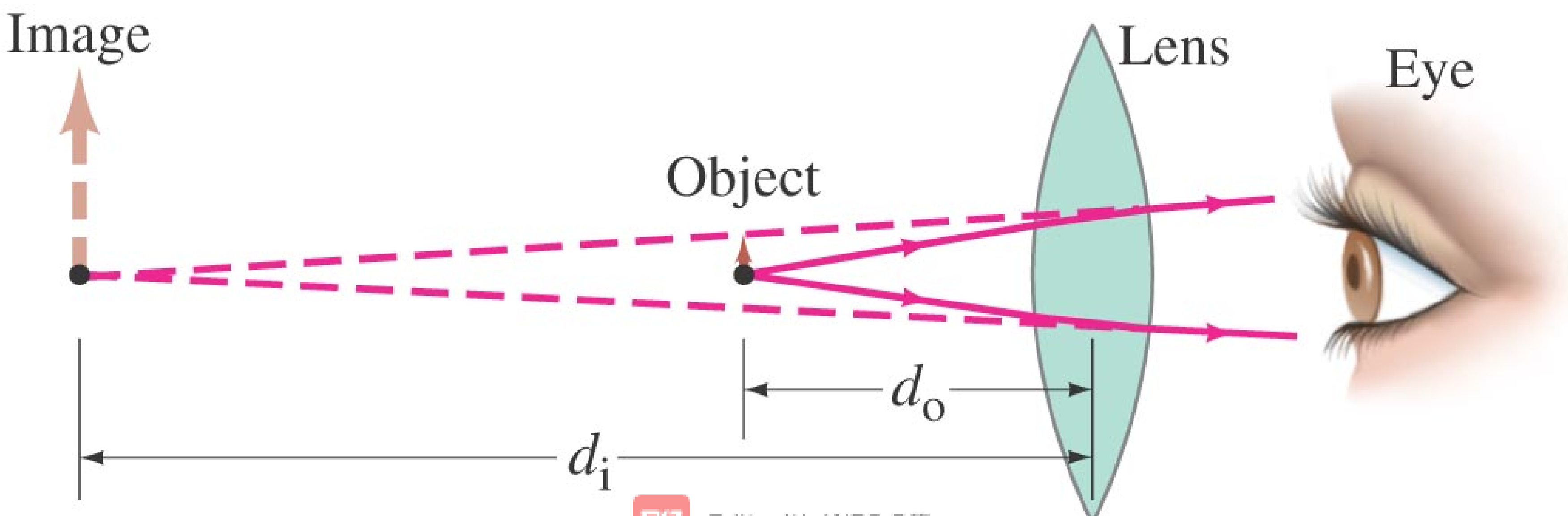
And farsightedness with a converging lens.



33-6 The Human Eye; Corrective Lenses

Example 33-12: Farsighted eye.

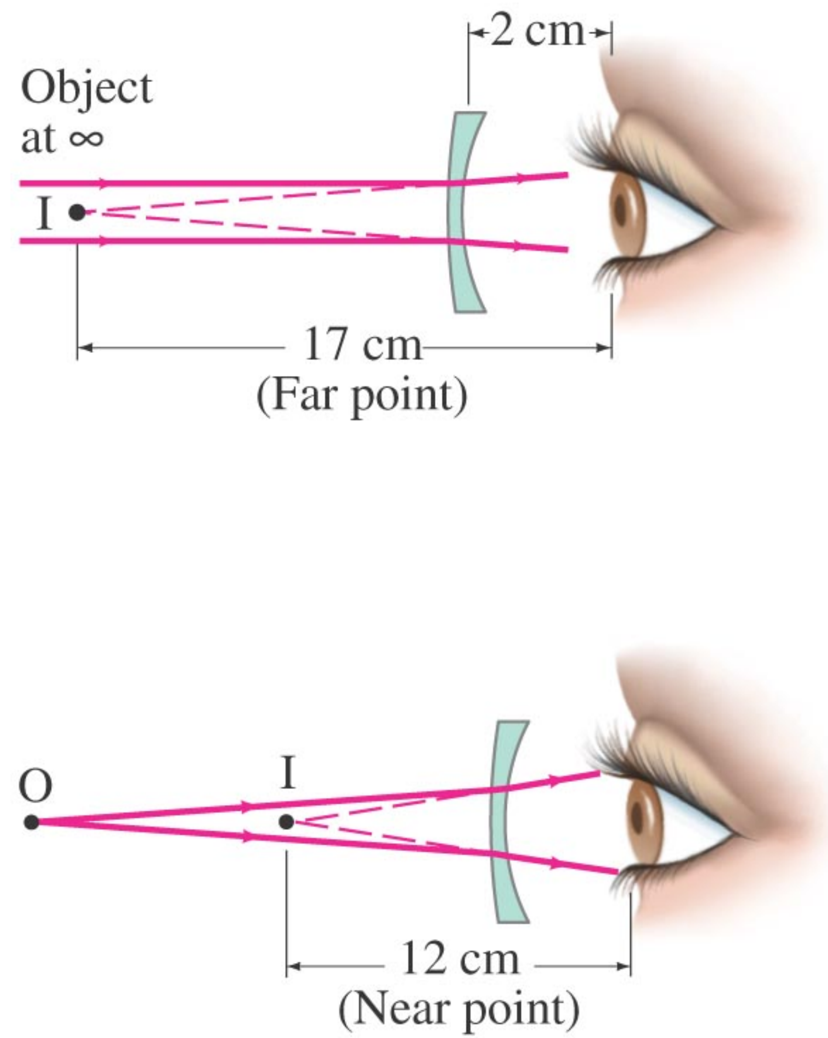
Sue is farsighted with a near point of 100 cm. Reading glasses must have what lens power so that she can read a newspaper at a distance of 25 cm? Assume the lens is very close to the eye.



33-6 The Human Eye; Corrective Lenses

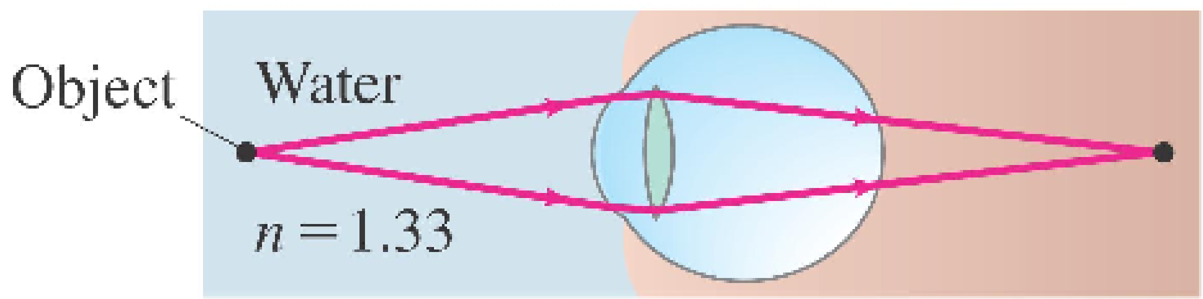
Example 33-13: Nearsighted eye.

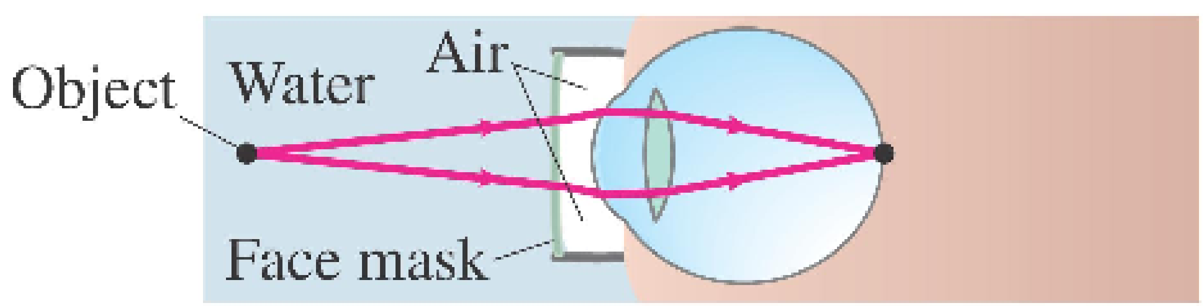
A nearsighted eye has near and far points of 12 cm and 17 cm, respectively. (a) What lens power is needed for this person to see distant objects clearly, and (b) what then will be the near point? Assume that the lens is 2.0 cm from the eye (typical for eyeglasses).



33-6 The Human Eye; Corrective Lenses

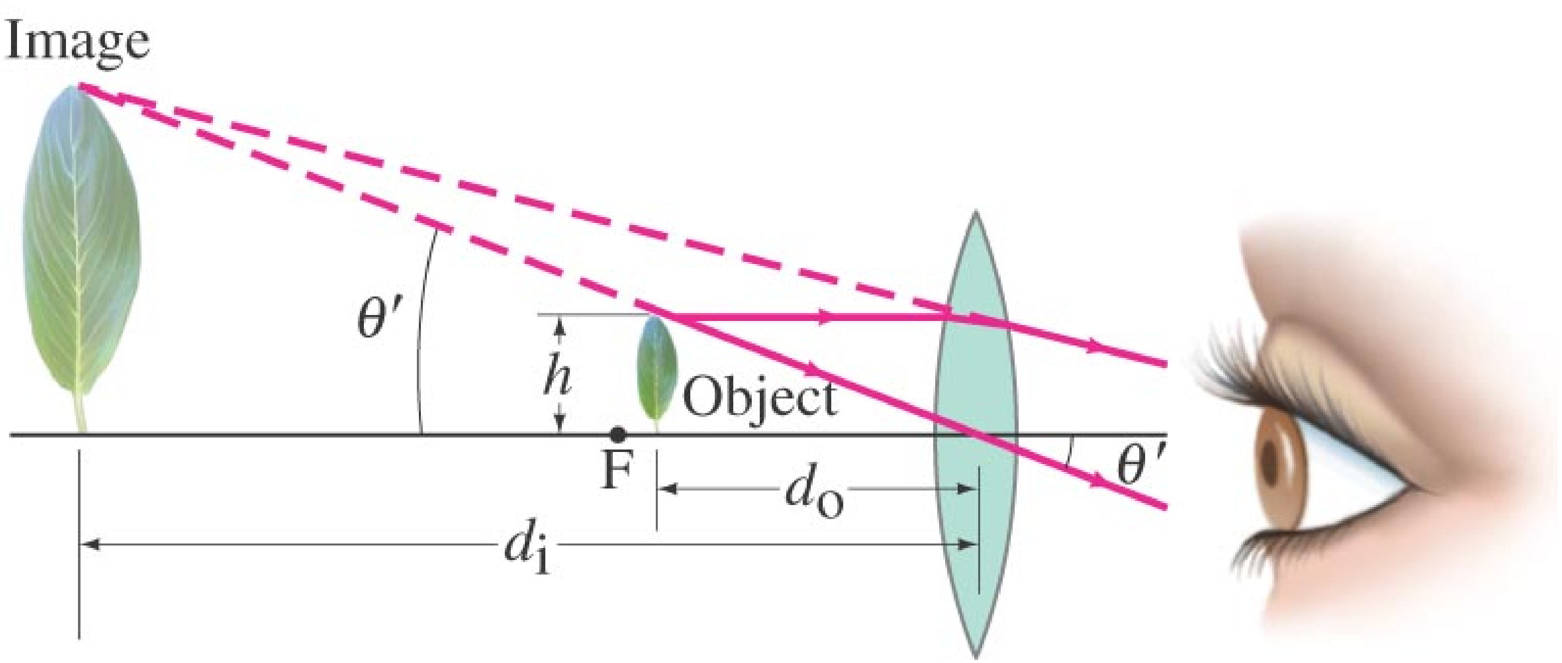
Vision is blurry under water because light rays are bent much less than they would be if entering the eye from air. This can be avoided by wearing goggles.





33-7 Magnifying Glass

A magnifying glass (simple magnifier) is a converging lens. It allows us to focus on objects closer than the near point, so that they make a larger, and therefore clearer, image on the retina.



33-7 Magnifying Glass

The power of a magnifying glass is described by its angular magnification:

$$M = \frac{\theta'}{\theta}.$$

If the eye is relaxed (*N* is the near point distance and *f* the focal length):

$$M = \frac{\theta'}{\theta} = \frac{h/f}{h/N} = \frac{N}{f}. \qquad \left[\begin{array}{c}\text{eye focused at } \infty; \\ N = 25 \text{ cm for normal eye}\end{array}\right]$$

If the eye is focused at the near point:

$$M = \frac{N}{f} + 1. \qquad \left[\begin{array}{c}\text{eye focused at near point, } N; \\ N = 25 \text{ cm for normal eye}\end{array}\right]$$

33-7 Magnifying Glass

Example 33-14: A jeweler's "loupe."

An 8-cm-focal-length converging lens is used as a "jeweler's loupe," which is a magnifying glass. Estimate (a) the magnification when the eye is relaxed, and (b) the magnification if the eye is focused at its near point $N = 25$ cm.

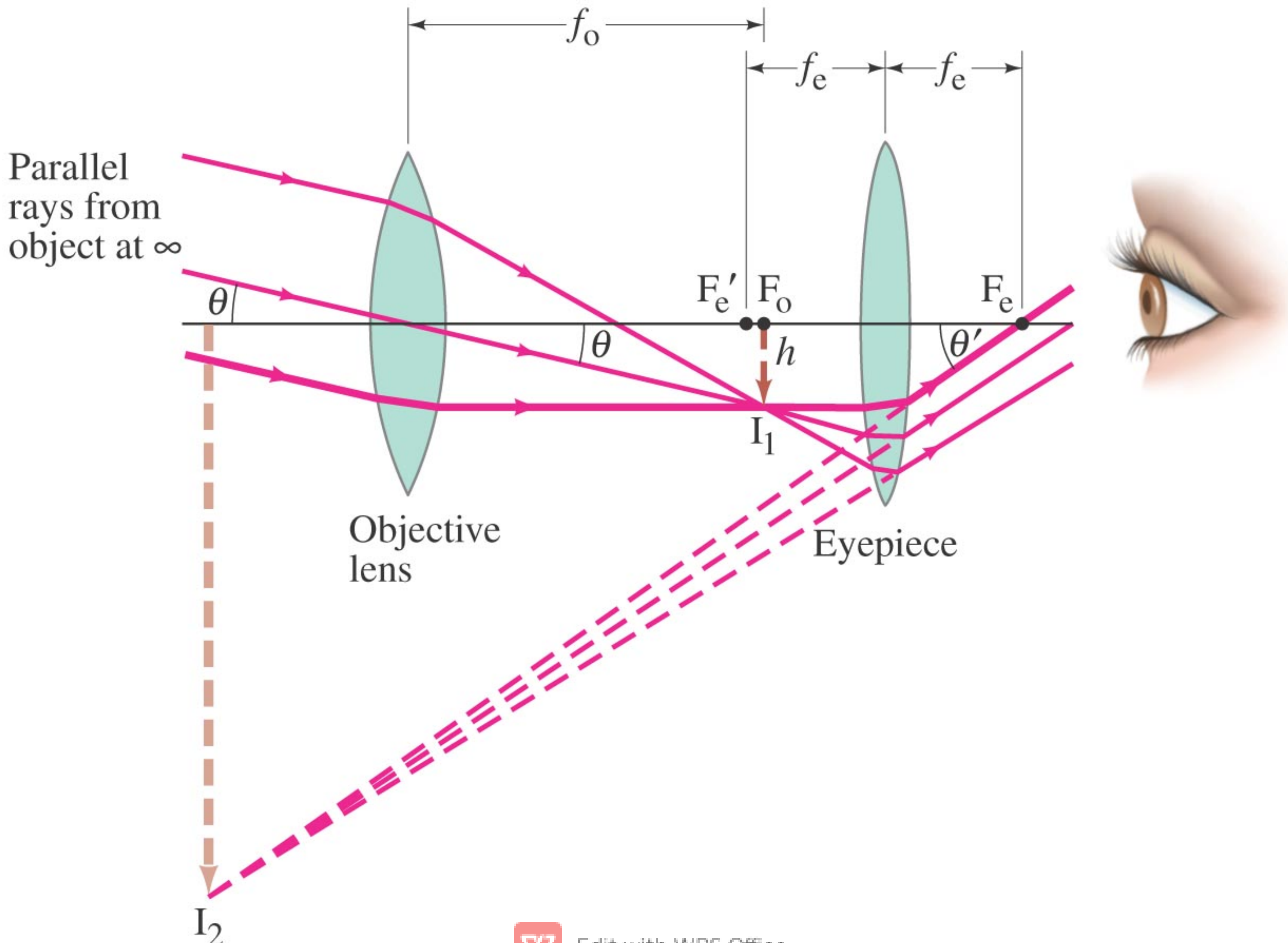
33-8 Telescopes

A refracting telescope consists of two lenses at opposite ends of a long tube. The objective lens is closest to the object, and the eyepiece is closest to the eye.

The magnification is given by

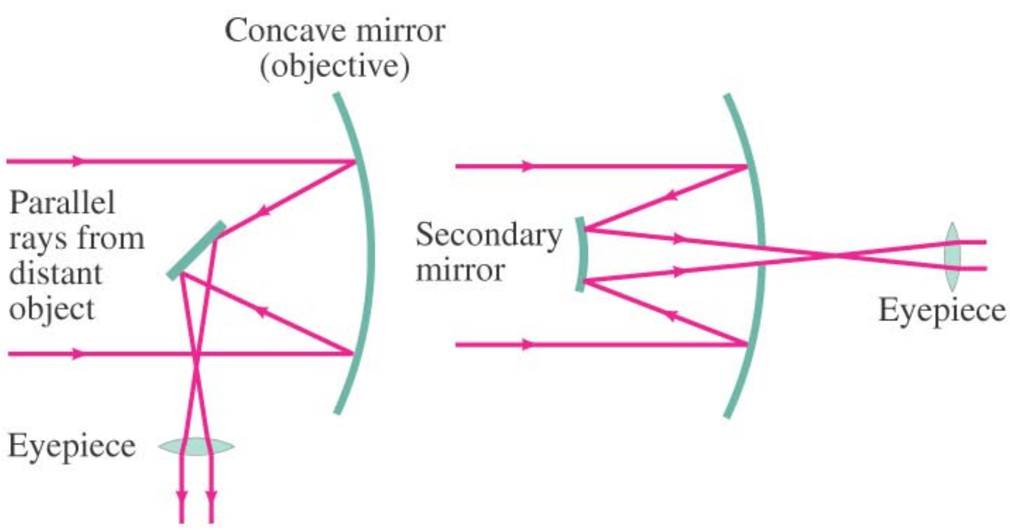
$$M = \frac{\theta'}{\theta} = \frac{(h/f_{\mathrm{e}})}{(h/f_{\mathrm{o}})} = -\frac{f_{\mathrm{o}}}{f_{\mathrm{e}}}. \qquad \text{[telescope]}$$

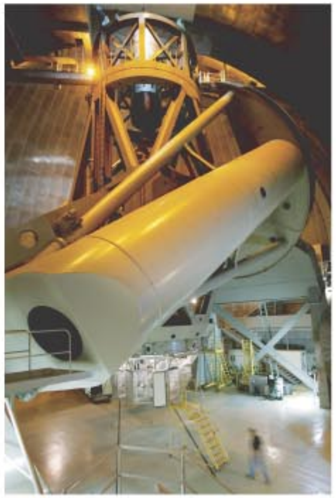
33-8 Telescopes



33-8 Telescopes

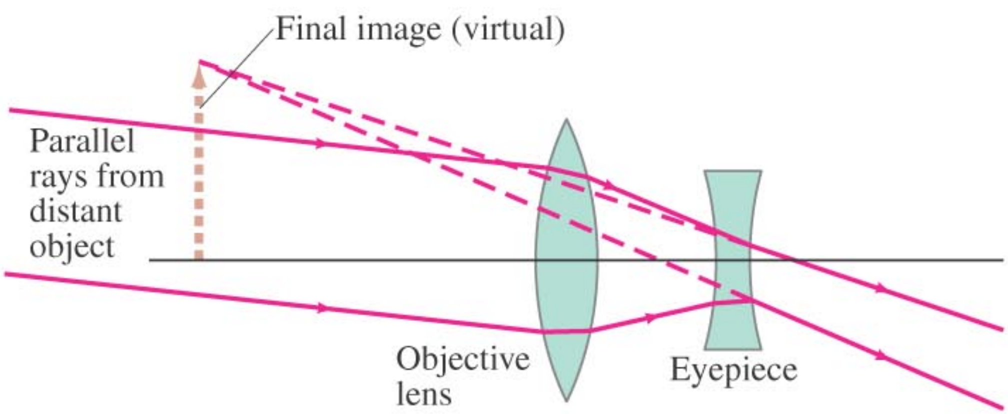
Astronomical telescopes need to gather as much light as possible, meaning that the objective must be as large as possible. Hence, mirrors are used instead of lenses, as they can be made much larger and with more precision.

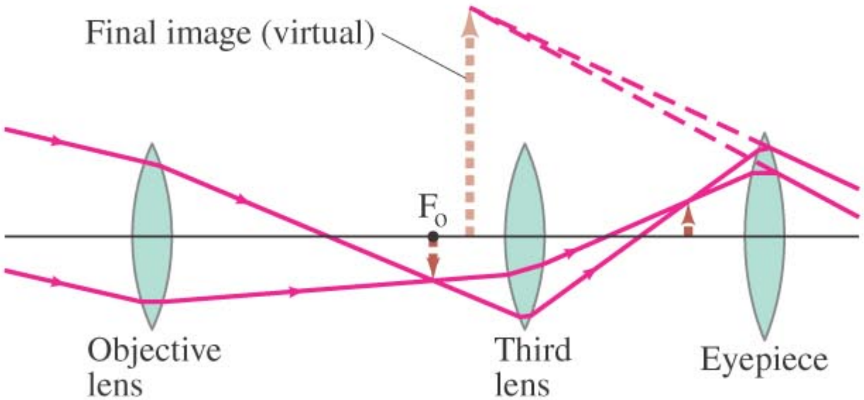




33-8 Telescopes

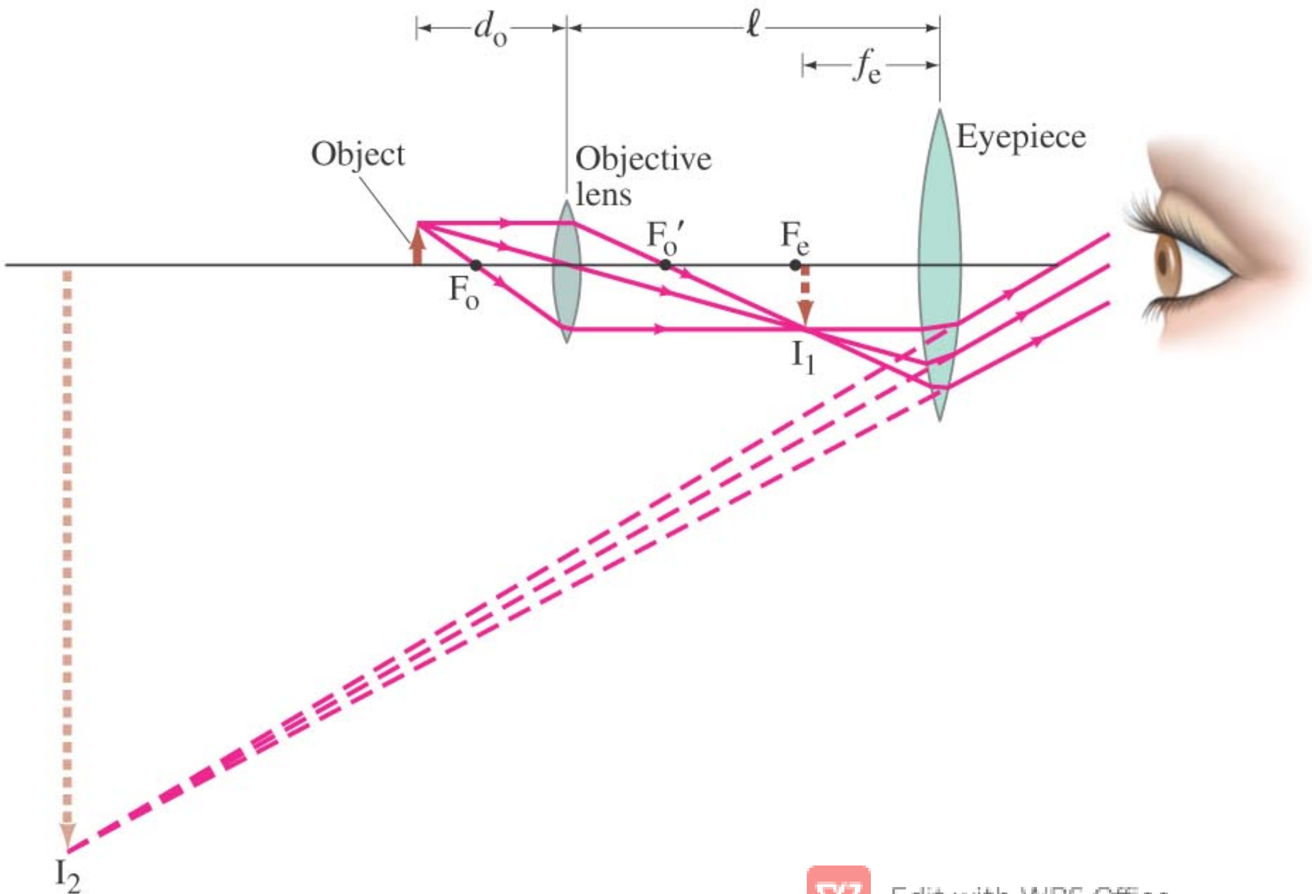
A terrestrial telescope, used for viewing objects on Earth, should produce an upright image. Here are two models, a Galilean type and a spyglass:





33–9 Compound Microscope

A compound microscope also has an objective and an eyepiece; it is different from a telescope in that the object is placed very close to the eyepiece.



33-9 Compound Microscope

The magnification is given by

$$M = M_e m_o = \left(\frac{N}{f_e}\right)\left(\frac{\ell - f_e}{d_o}\right) \qquad \text{[microscope]}$$

$$\approx \frac{N\ell}{f_e f_o}. \qquad [f_o \text{ and } f_e \ll \ell]$$

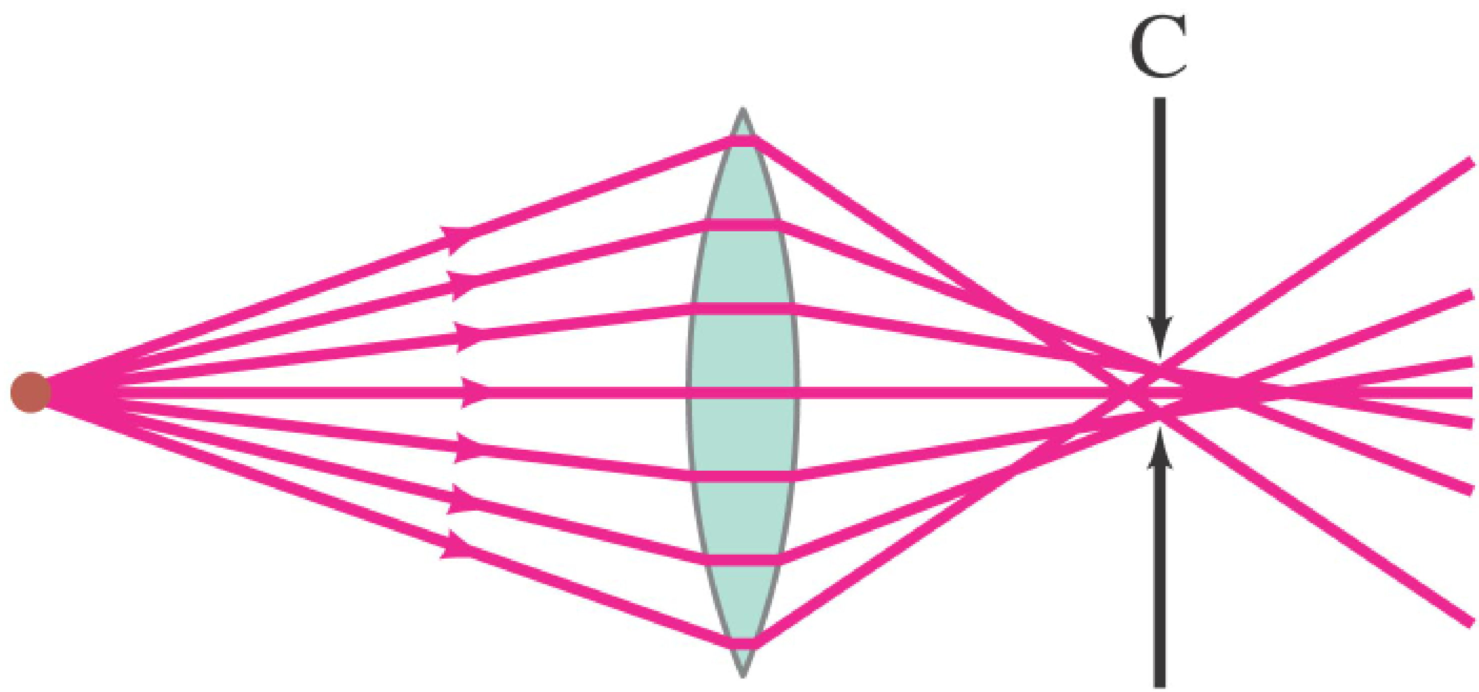
33-9 Compound Microscope

Example 33-16: Microscope.

A compound microscope consists of a 10X eyepiece and a 50X objective 17.0 cm apart. Determine (a) the overall magnification, (b) the focal length of each lens, and (c) the position of the object when the final image is in focus with the eye relaxed. Assume a normal eye, so $N = 25$ cm.

33-10 Aberrations of Lenses and Mirrors

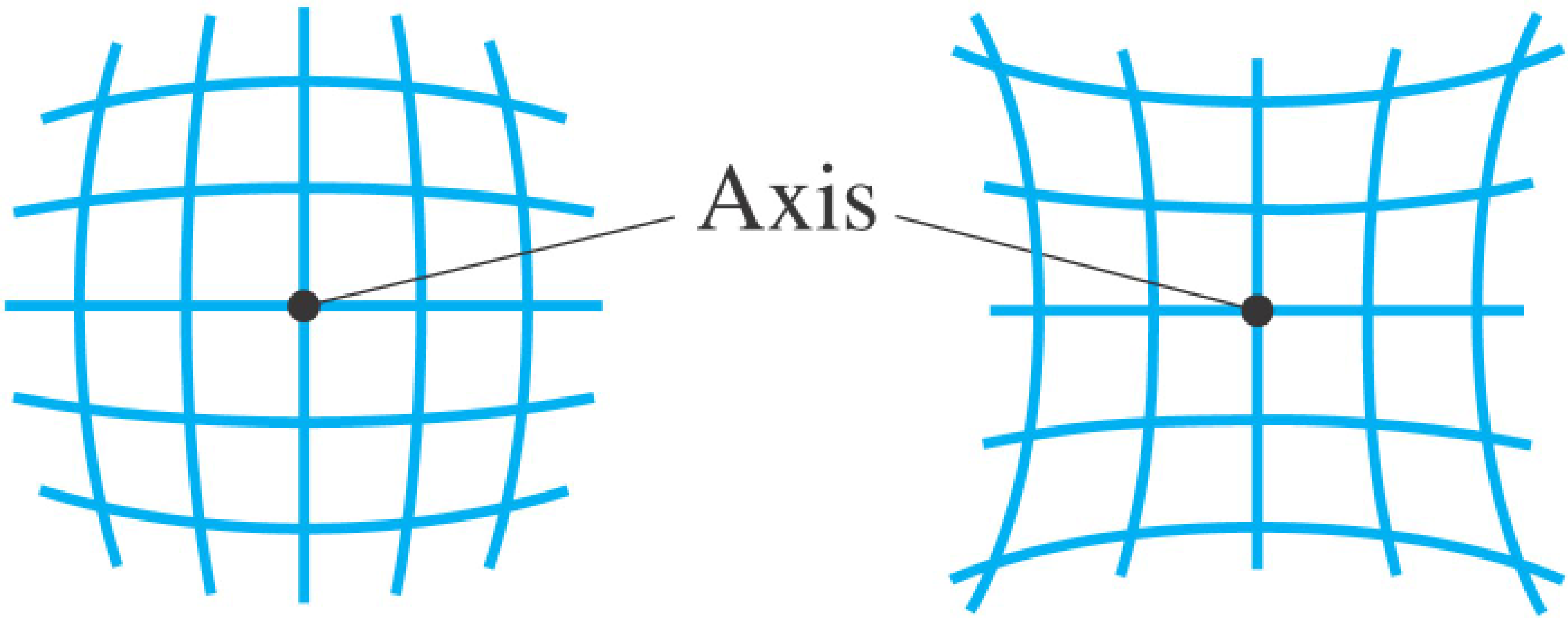
Spherical aberration: rays far from the lens axis do not focus at the focal point.



Solutions: compound-lens systems; use only central part of lens.

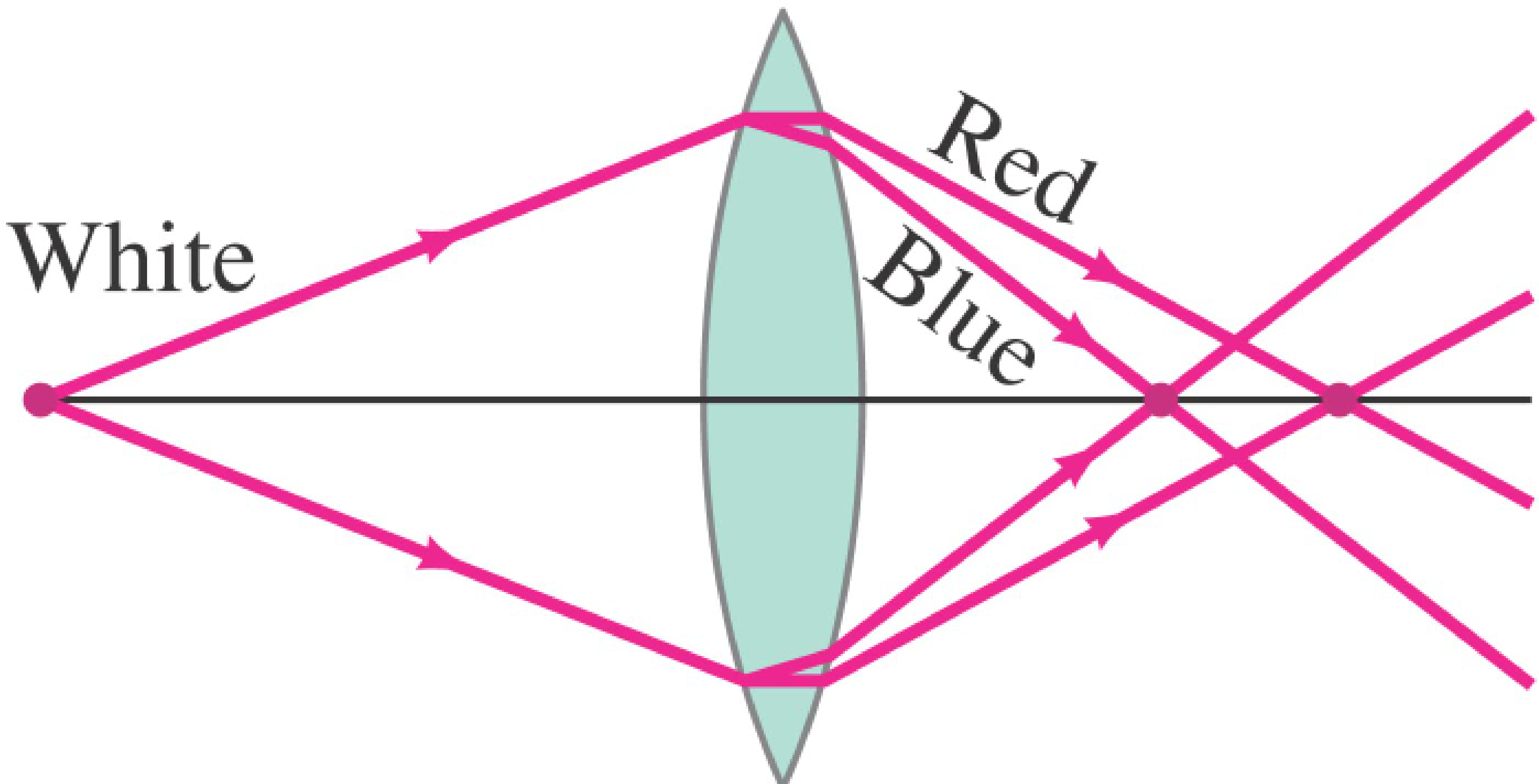
33-10 Aberrations of Lenses and Mirrors

Distortion: caused by variation in magnification with distance from the lens. Barrel and pincushion distortion:



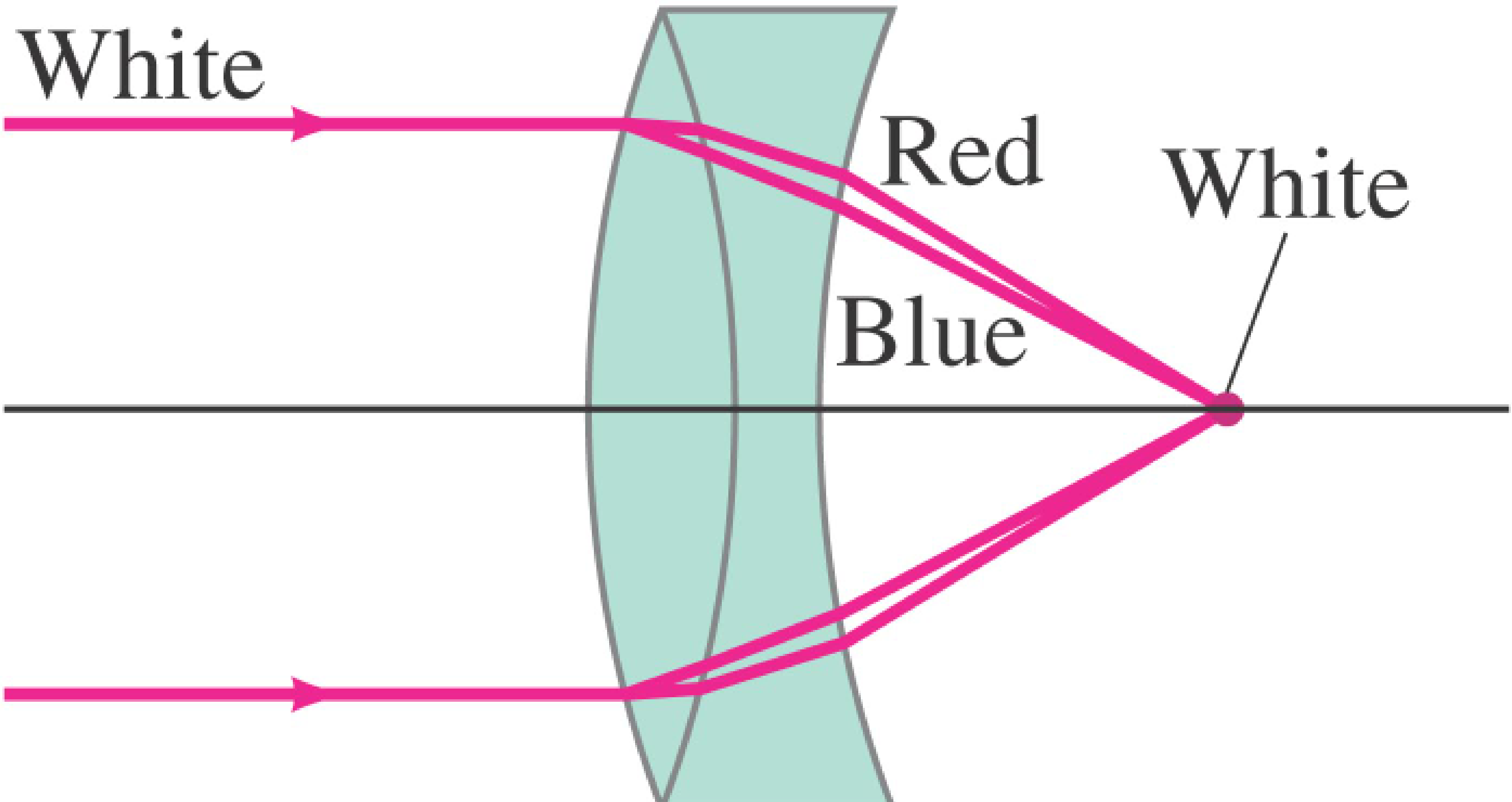
33-10 Aberrations of Lenses and Mirrors

Chromatic aberration: light of different wavelengths has different indices of refraction and focuses at different points.



33-10 Aberrations of Lenses and Mirrors

Solution: Achromatic doublet, made of lenses of two different materials



Summary of Chapter 33

- Lens uses refraction to form real or virtual image.
- Converging lens: rays converge at focal point.
- Diverging lens: rays appear to diverge from focal point.
- Power is given in diopters (m^{-1}):

$$P = \frac{1}{f}.$$

Summary of Chapter 33

- Thin lens equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.$$

- Magnification:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}.$$

Summary of Chapter 33

- Camera focuses image on film or electronic sensor; lens can be moved and size of opening adjusted (*f*-stop).
- Human eye also makes adjustments, by changing shape of lens and size of pupil.
- Nearsighted eye is corrected by diverging lens.
- Farsighted eye is corrected by converging lens.

Summary of Chapter 33

- Magnification of simple magnifier:

$$M = \frac{\theta'}{\theta} = \frac{h/f}{h/N} = \frac{N}{f}. \qquad \left[\begin{array}{c}\text{eye focused at } \infty; \\ N = 25 \text{ cm for normal eye}\end{array}\right]$$

- Telescope: objective lens or mirror plus eyepiece lens. Magnification:

$$M = \frac{\theta'}{\theta} = \frac{(h/f_e)}{(h/f_o)} = -\frac{f_o}{f_e}. \qquad \text{[telescope]}$$