Current Electricity

Outline

- Electric current,
- potential difference,
- resistance and resistivity,
- Ohm's law, Ohmic and non-ohmic conductors,
- resistors in series and in parallel,
- electromotive force and circuit,
- electrical energy,
- electrical power and efficiency

Electric Current

The **electric current** is defined as the charge (ΔQ) passing through a given cross-sectional area A of a wire per unit time, that is,

$$I = \frac{\Delta Q}{\Delta t}$$

The unit of current is Coulomb per second, which is called ampere (A).

$$I = \frac{dQ}{dt} \qquad \text{differential form}$$

The total charge Q that passes through area A in a time t is $Q = qnAv_d t$
where n = number of charges q per unit volume and v_d = drift velocity of the charges

Electric Current

$$I = \frac{Q}{t} = qnAv_d$$

The **current density** is defined as electric current per unit cross-sectional area. For a uniform current flow, we have,

$$J = \frac{I}{A} = qnv_d$$

The S.I. unit of *J* is ampere per metre square (Am^{-2}).

Ex 1: A copper wire has a square cross section, 2.0 mm on a side. It carries a current of 10 A. If the density of free electron is 8 x 10^{28} m^{-2}, calculate the current density in the wire and the drift speed.

$$J = \frac{I}{A} = \frac{10}{4 \times 10^{-6}} Am^{-2} = 2.5 \times 10^{6} Am^{-2}$$

$$v = \frac{J}{nq} = \frac{2.5 \times 10^{6}}{8 \times 10^{28} \times 1.6 \times 10^{-19}} m/s = 1.953 \times 10^{-4} m/s$$

Ohm's Law

Ohm's law states that under constant physical conditions, the current in a conducting wire is proportional to the potential difference V applied to its ends.

$I \alpha V$ which implies $I = \dfrac{V}{R}$

It is found experimentally that the resistance R of a wire is proportional to the length of the wire and inversely proportional to the cross-sectional area A, that is

$R \alpha \dfrac{L}{A}$ which implies $R = \rho \dfrac{L}{A}$

where ρ is the constant of proportionality which is known as the **resistivity** of the material. The resistivity ρ depends only on the material and the temperature. The unit of resistivity ρ is ohm-metre (Ωm).

Conductivity, σ, is the reciprocal of the resistivity, that is

$$\sigma = \frac{1}{\rho} \quad (\Omega \text{m})^{-1}$$

Ohmic and Non Ohmic Conductors

Ohmic conductors are conductors that obey Ohm's law.

-the characteristic or *I*–*V* graph is a straight line passing through the origin.

For example, copper and tungsten obey Ohm's law.

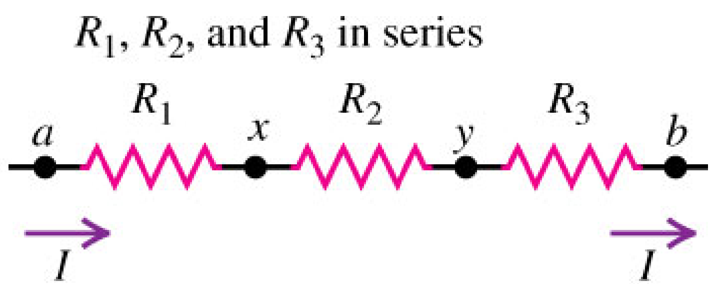
Non-ohmic conductors are those which do not obey Ohm's law.

For example, junction diode, neon gas, diode valve, dilute H_2SO_4 (platinum electrodes) do not obey Ohm's law.

A non-ohmic characteristic or *I*–*V* graph may have a curve instead of a straight line; or it may not pass through the origin.

Resistors in series and parallel

- Resistors are in ***series*** if they are connected one after the other so the current is the same in all of them.
- The *equivalent resistance* of a series combination is the *sum* of the individual resistances: $R_{eq} = R_1 + R_2 + R_3 + \ldots$



Series Resistors have resistance LARGER than the largest value present.

i. The same current flows through all resistors in series

ii. Total potential difference = sum of individual potential differences
$V = V_1 + V_2 + V_3$

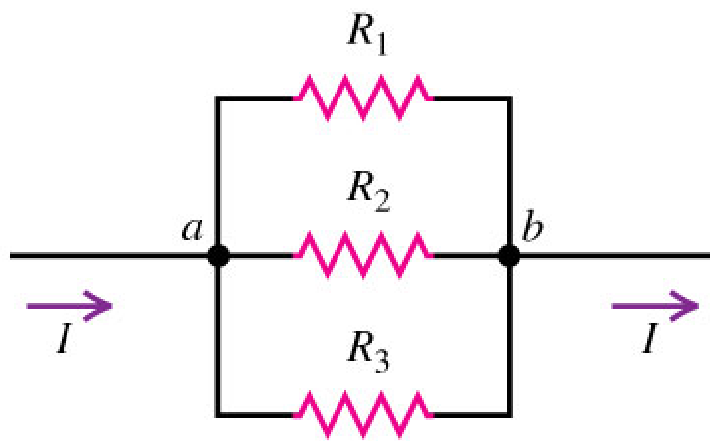
iii. Individual potential differences is directly proportional to individual resistances

$V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$

Resistors in series and parallel

- Resistors are in ***parallel*** if they are connected so that the potential difference must be the same across all of them.
- The *equivalent resistance* of a parallel combinaton is given by $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$

R_1, R_2, and R_3 in parallel



Parallel Resistors have resistance SMALLER than the smallest value present.

i. Total current, I is equal to sum of individual currents flowing through the resistors in parallel; $I = I_1 + I_2 + I_3$

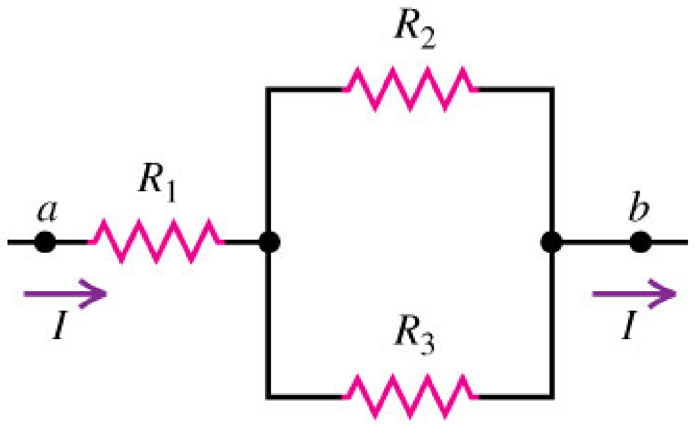
ii. The potential difference across each resistor is the same and is equal to the full voltage V.
iii. Individual current is inversely proportional to individual resistances

$V = I_1R_1$, $V = I_2R_2$ and $V = I_3R_3$

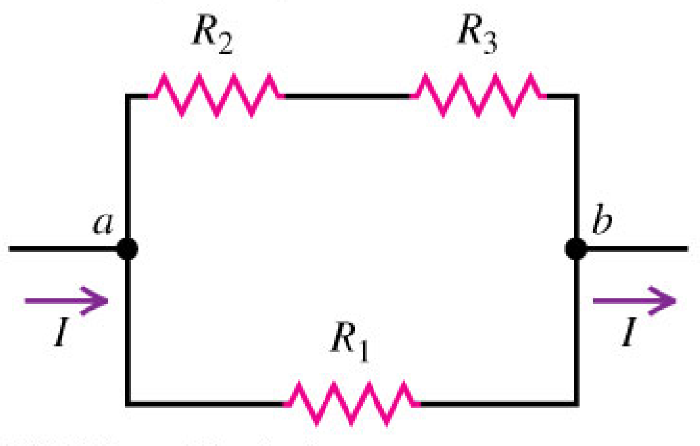
Series and parallel combinations

- Resistors can be connected in combinations of series and parallel

(c) R_1 in series with parallel combination of R_2 and R_3



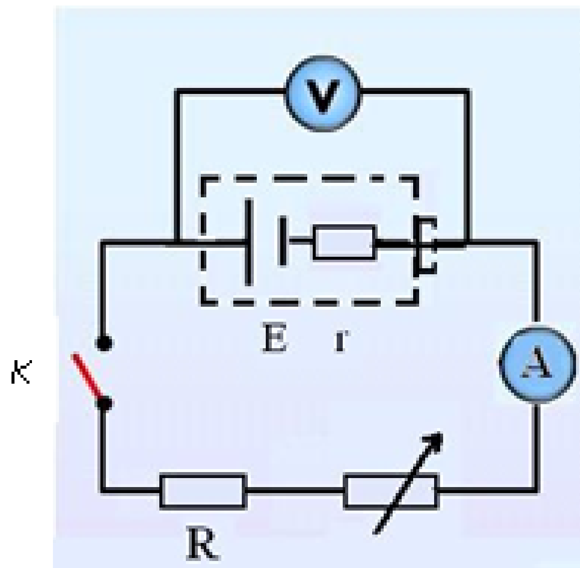
(d) R_1 in parallel with series combination of R_2 and R_3



Electromotive force and Circuit

- The **electromotive force** or e.m.f. (E) is the potential difference across the terminals of a battery (or any other generator) on open circuit, that is, when no current is flowing.
- The e.m.f. of a battery depends on the nature of the chemicals used and not on its size.
- However, the internal resistance, *r*, of the battery depends on the size of the battery.

Electromotive force and Circuit



$$I = \frac{E}{R + r}$$

Terminal p.d. , $V = IR = \dfrac{ER}{R + r}$

Example 2: A battery of e.m.f. 4V and internal resistance 2 Ω is joined to a resistor of 8 Ω. Calculate the terminal p.d.

$$I = \frac{E}{R + r} = \frac{4}{8 + 2} A = 0.4\, A$$

$V = IR = 0.4 \times 8$ V = 3.2V

Electrical Power and Efficiency

The electrical work (W) required to transfer a charge q through a potential difference V is given by

$$W = qV$$

Electrical Power $P = \dfrac{Electrical \quad Work}{Time} = \dfrac{qV}{t} = VI$

The unit of power is watt (1W = 1J/s).

We can also define the e.m.f. E of a battery or any other generator as the total energy per coulomb it delivers round a circuit joined to it.

Total electrical power generated is

$$P = \frac{W}{t} = EI$$

Electrical Power and Efficiency

Efficiency, η, of a circuit is the ratio of the power output to the power generated:

$$\eta = \frac{Power \quad output}{Power \quad generated}$$

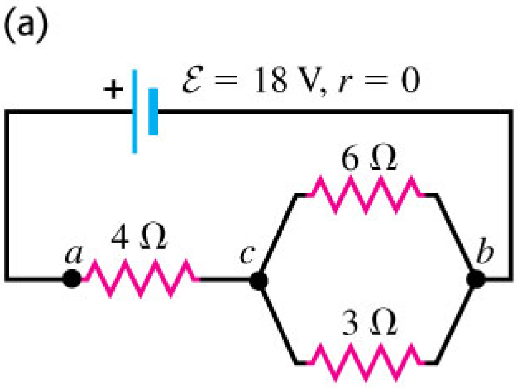
$$\eta = \frac{P_{out}}{P_{gen}} = \frac{IV}{IE} = \frac{V}{E}$$

$$\eta = \frac{R}{R + r}$$

How come?

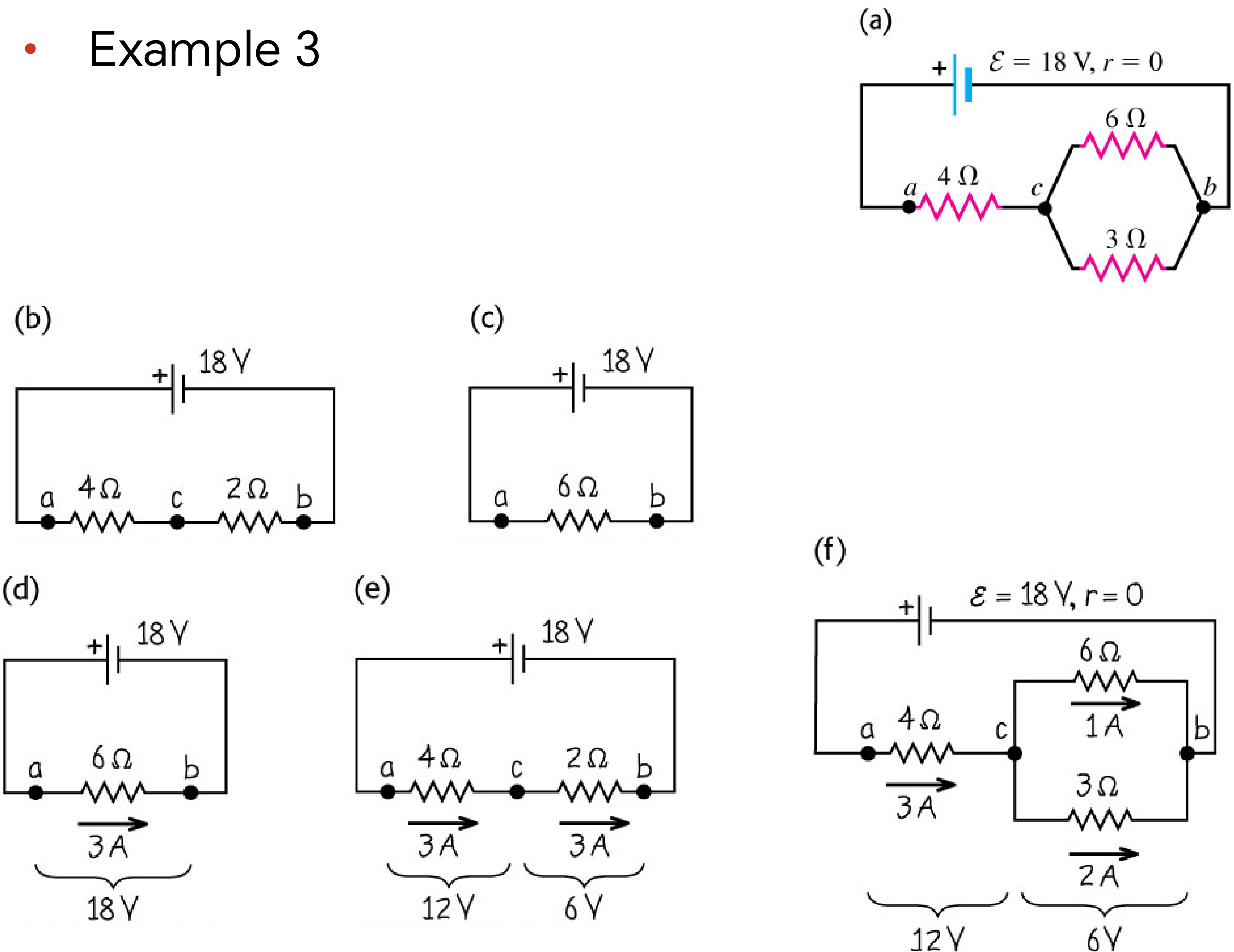
Equivalent resistance

- Example 3



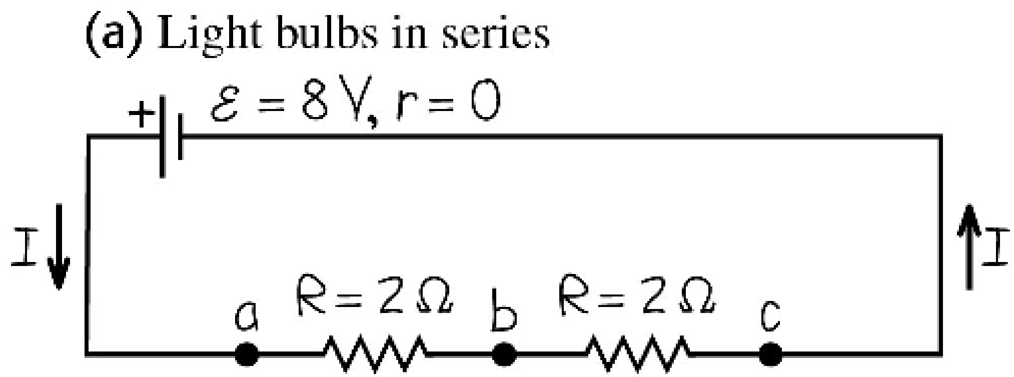
Equivalent resistance

- Example 3



Series versus parallel combinations

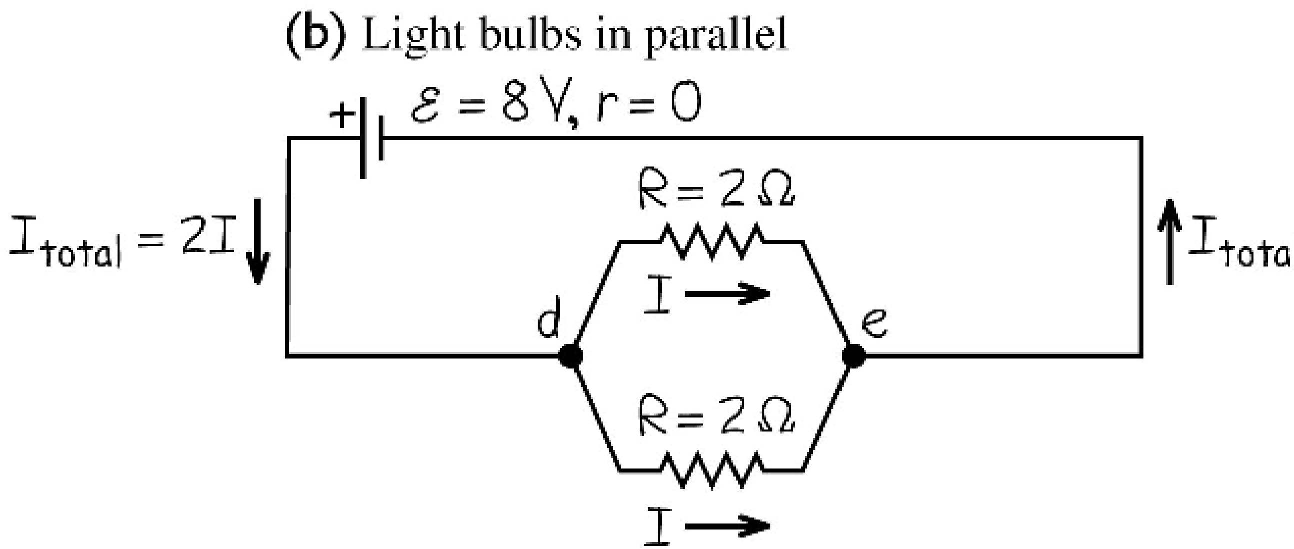
- Ex 4: Current through each R & Power dissipated?



- $R_{equivalent} = 2 + 2 = 4$ Ohms
- $I = 8V / 4\ \Omega = 2\ A$
- Power = i^2R = 16 Watts total (8 Watts for each bulb)

Series versus parallel combinations

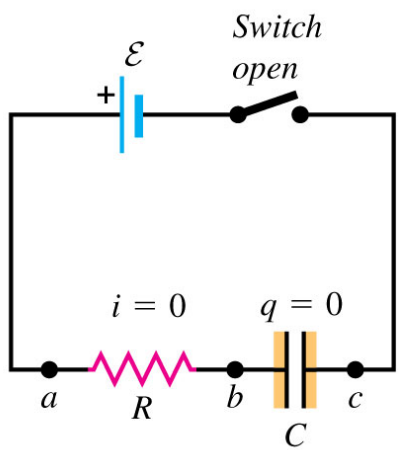
- Ex 5: Current through each R & Power dissipated?



- $R_{equivalent} = (½ + ½)^{-1} = 1$ Ohm
- I = 8V / 1 $\Omega = 8$ A
- Power = i^2R = 64 Watts total (32 Watts for each bulb)

Adding Capacitors to DC circuits!

- **RC** circuits include
  - Batteries (Voltage sources!)
  - Resistors
  - Capacitors
  - … and switches!

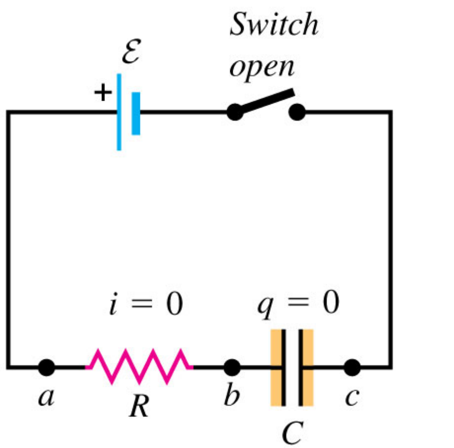


- RC circuits will involve **TIMING** considerations
  - Time to *fill up* a capacitor with charge
  - Time to *drain* a capacitor that is already charged

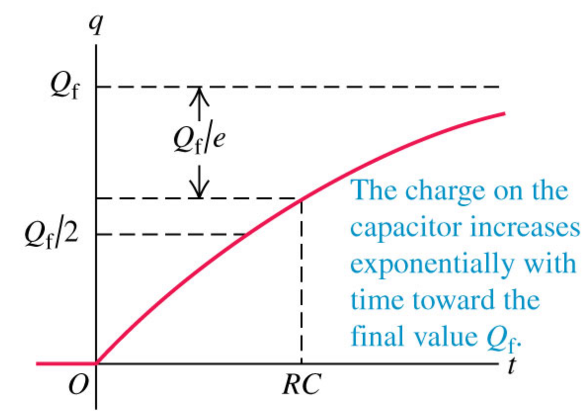
Charging a capacitor

- Start with uncharged capacitor! What happens??

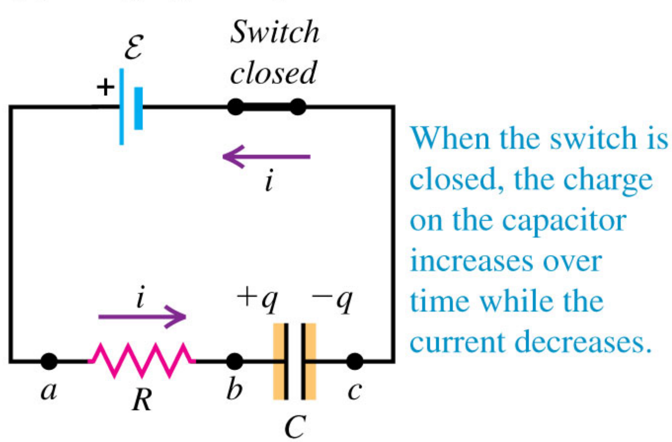
(a) Capacitor initially uncharged



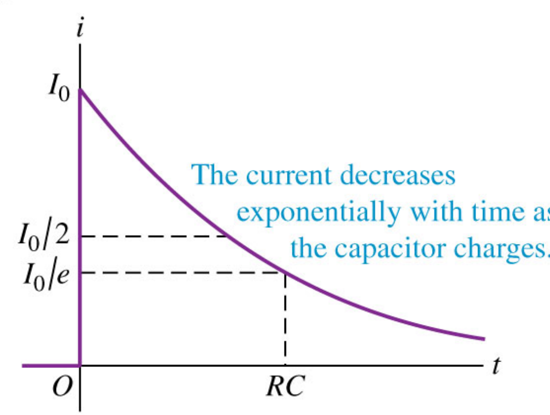
(b) Graph of capacitor charge versus time for a charging capacitor



(b) Charging the capacitor

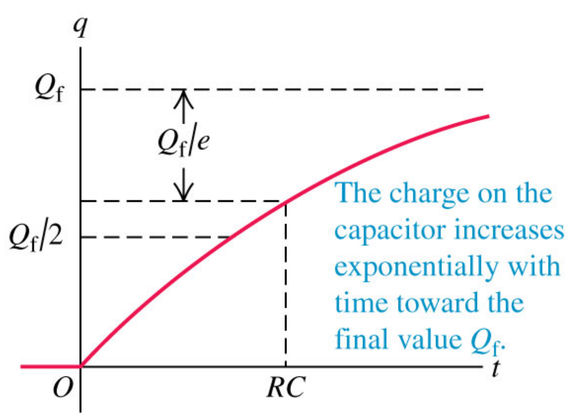


(a) Graph of current versus time for a charging capacitor



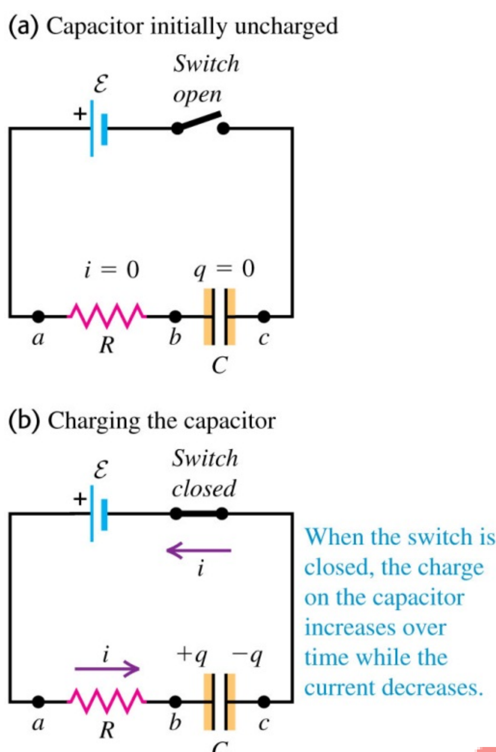
Adding Capacitors to DC circuits!

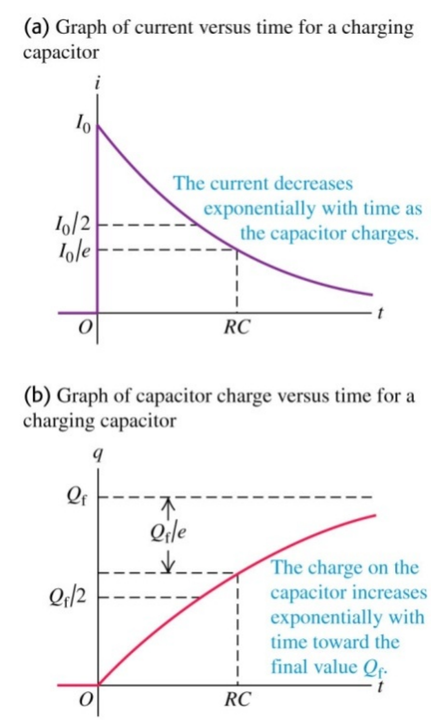
- **In charging RC** circuits the *time constant* is τ
- τ *increases* with R
    - Larger resistors decrease current
    - Less charge/time arrives at capacitor
    - It takes longer to fill up capacitor
- τ *increases* with C
    - Larger capacitors have more capacity!
    - They take longer to fill up!



Charging a capacitor

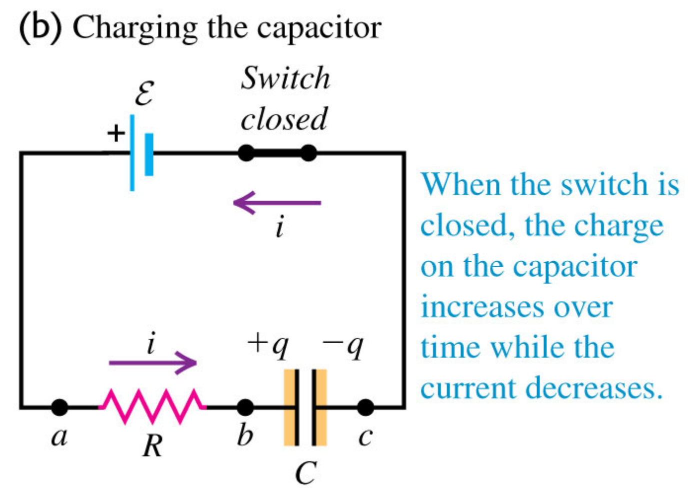
- The *time constant* is $\tau = RC$.
- In ONE time constant:
  - Current drops to 1/e of initial value (about 36%)
  - Charge on capacitor plates rises to ~64% of maximum value





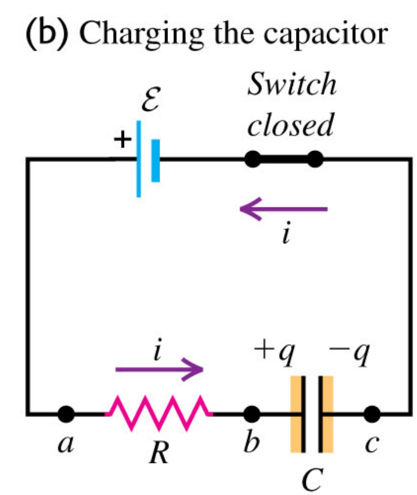
Charging a Capacitor

- $V_{ab} = iR$
- $C = q/V_{bc}$ so $V_{bc} = q/C$
- $E - iR - q/C = 0$
    - Current is a function of time
    - $i = dq/dt$
    - $E - (dq/dt)R - q(t)/C = 0$
    - *A differential equation involving charge q*



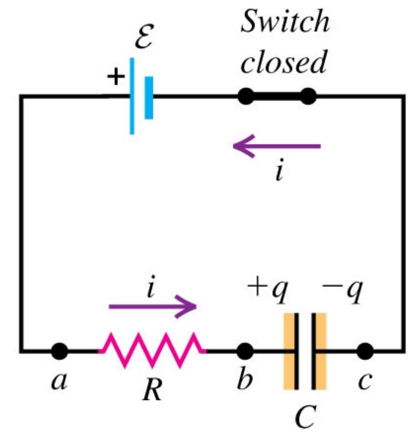
Charging a Capacitor

- $E - (dq/dt)R - q/C = 0$
- *Boundary conditions* relate q and t at key times:
  - q(t) on capacitor = 0 @ t=0
  - q= Qmax, i(t) = 0 @ t = ∞ (when capacitor is full)



Charging a Capacitor

- $E - iR - q/C = 0$
- General Solution
  - $q(t) = Q_{max}\,(1 - e^{-t/RC})$
  - Check?



    - q(t) on capacitor = 0 @ t=0
    - $i(t) = dq/dt = Q_{max}/RC\; e^{-t/RC}$
    - i(0) is maximum current, $Qmax/RC = E/R = I_0$
    - $i(t) = 0$ when capacitor is full, @ $t = \infty$

Charging a capacitor

- 10 MΩ resistor connected in series with 1.0 μF un-charged capacitor and a battery with emf of 12.0 V.
  - What is time constant?
  - What fraction of final charge is on capacitor after 46 seconds?
  - What fraction of initial current I0 is flowing then?

Charging a capacitor

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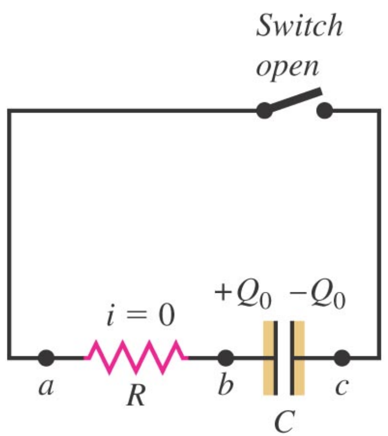
  - What is time constant?
  - What fraction of final charge is on capacitor after 46 seconds?
  - What fraction of initial current I_0 is flowing then?

- $\tau = RC = 10$ seconds
- $Q(t) = Q_{max}(1 - e^{-t/RC})$ so $Q(_{46\ \text{seconds}})/Qmax = 99\%$
- $I(t) = I_0\, e^{-t/RC}$ so $I(_{46\ \text{seconds}})/I_0 = 1\%$

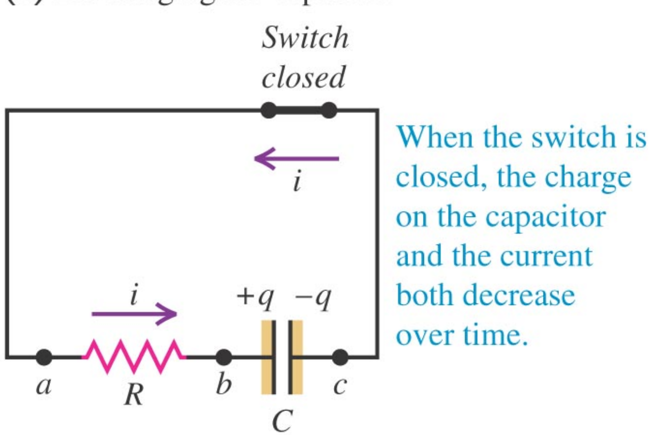
Discharging a capacitor

- Disconnect Battery – let Capacitor “drain”

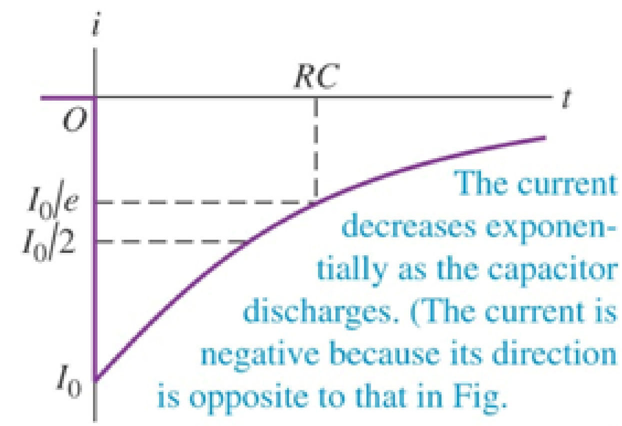
(a) Capacitor initially charged



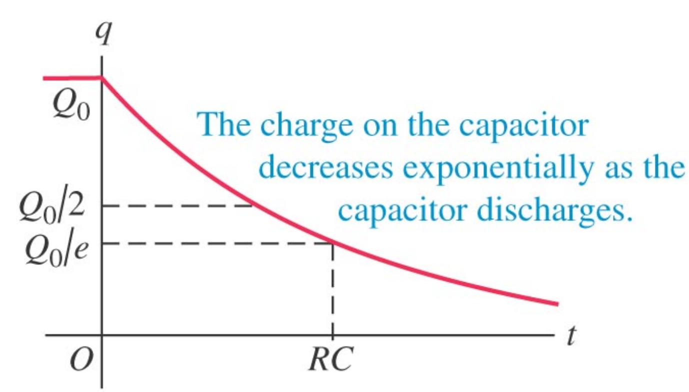
(b) Discharging the capacitor



(a) Graph of current versus time for a discharging capacitor

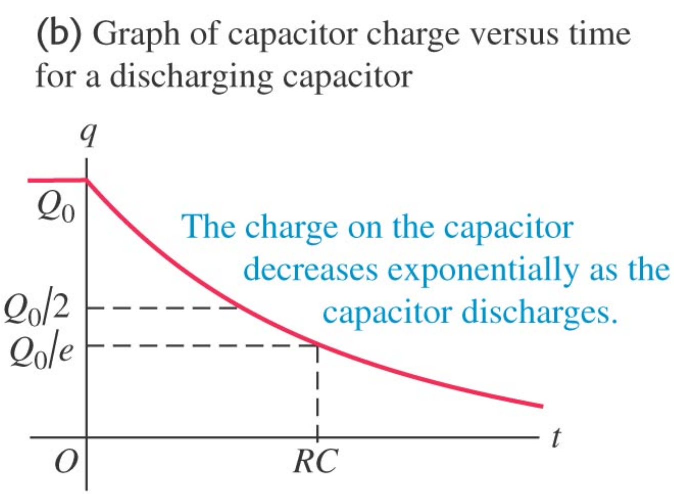


(b) Graph of capacitor charge versus time for a discharging capacitor



Adding Capacitors to DC circuits!

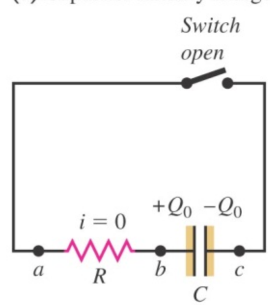
- **In discharging RC** circuits the *time constant* is still τ
- τ *increases* with R
    - Larger resistors decrease current
    - Less charge/time *leaves* capacitor
    - It takes longer to *drain* capacitor
- τ *increases* with C
    - Larger capacitors have more capaci
    - They take longer to drain!



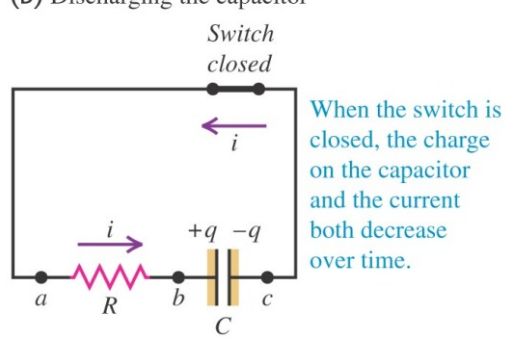
Discharging a capacitor

- Time constant is still RC!
- In one time time constant:
  - Charge on plates DROPS by 64%
  - Current through resistor DROPS by 64% (“negative”)

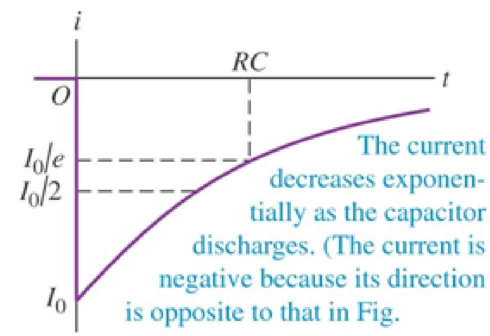
(a) Capacitor initially charged



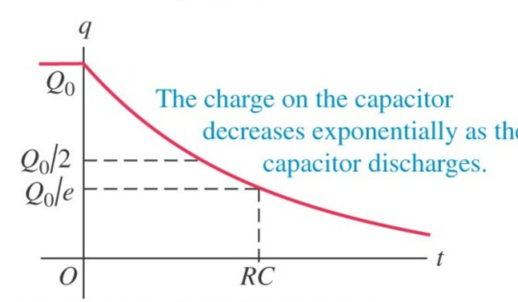
(b) Discharging the capacitor



(a) Graph of current versus time for a discharging capacitor

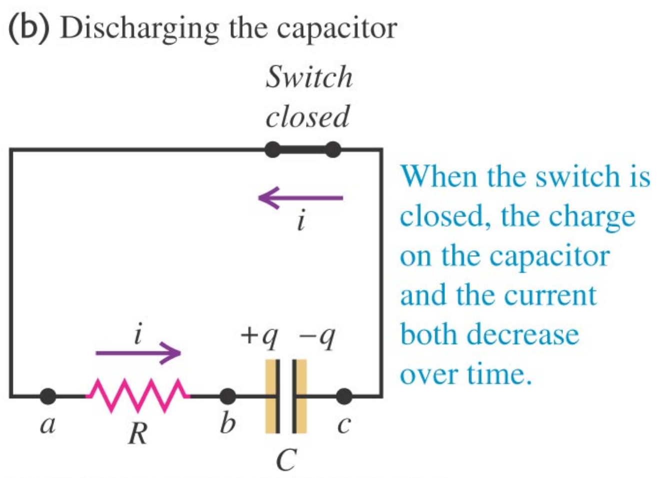


(b) Graph of capacitor charge versus time for a discharging capacitor



Charging a Capacitor

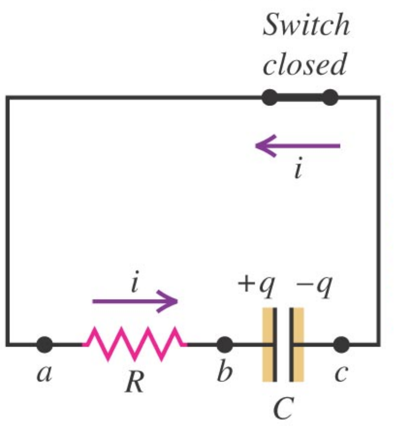
- $V_{ab} = iR$
- $V_{bc} = q/C$
- $iR + q/C = 0$



  - Another differential equation involving charge
  - *$i = dq/dt$*
  - *$(dq/dt)R + q(t)/C = 0$*

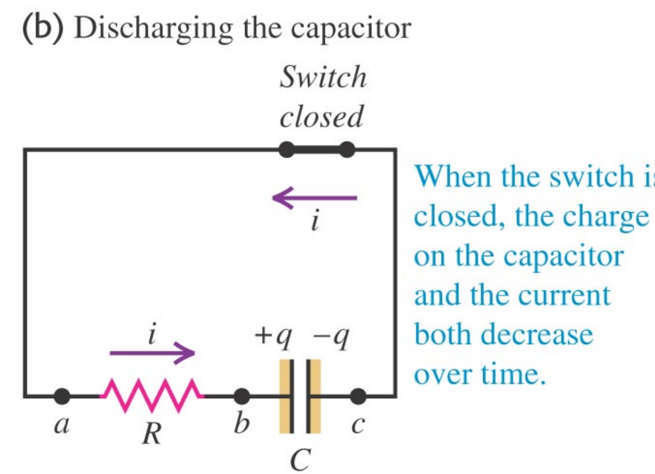
Charging a Capacitor

- $iR + q/C = 0$
- $(dq/dt)R + q(t)/C = 0$
- Boundary conditions:
    - q(t) on capacitor = Qmax
      @ t=0
    - q= 0, i(t) = 0
      @ t = ∞
      when capacitor is *empty*



Charging a Capacitor

- $iR + q/C = 0$
- General Solution
  - $q(t) = Q_{max}\, e^{-t/RC}$
    - $i = dq/dt = I_0 e^{-t/RC}$
    - Check
      - q(t) on capacitor = Qmax @ t=0
      - q= 0, i(t) = 0 when capacitor is empty, @ t = ∞



Discharging a capacitor

- Same circuit as before; 10 MΩ resistor connected in series with 1.0 μF capacitor; battery with emf of 12.0 V is disconnected.
- Assume at $t = 0$, $Q(0) = 5.0\ \mu C$.
- When will charge = 0.50 μC?
- What is current then?

Discharging a capacitor

- Same circuit as before; 10 MΩ resistor connected in series with 1.0 μF capacitor; battery with emf of 12.0 V is disconnected.
- Assume at $t = 0$, $Q(0) = 5.0\ \mu C$.
- When will charge = 0.50 μC?
- What is current then?

- $\tau = RC = 10$ seconds (still!)
- $Q(t) = Q_{max}\, e^{-t/RC}$ so $Q(t) = 1/10^{th}$ Q max =>
  $t = RC \ln(Q/Qmax) = 23$ seconds $(2.3\ \tau)$
- $I(t) = -Q_0/RC\,(e^{-t/RC})$ so $I(2.3\ \tau) = -5.0 \times 10^{-8}$ Amps

Assignment (Submission deadline – 08/04/2015; 2pm)

- **13.1.** Figure 13.19 shows a battery of 12 V and negligible internal resistance connected to three resistors. Calculate the p.d. V across the 6 Ω resistor and the current passing through the 8 Ω resistor.

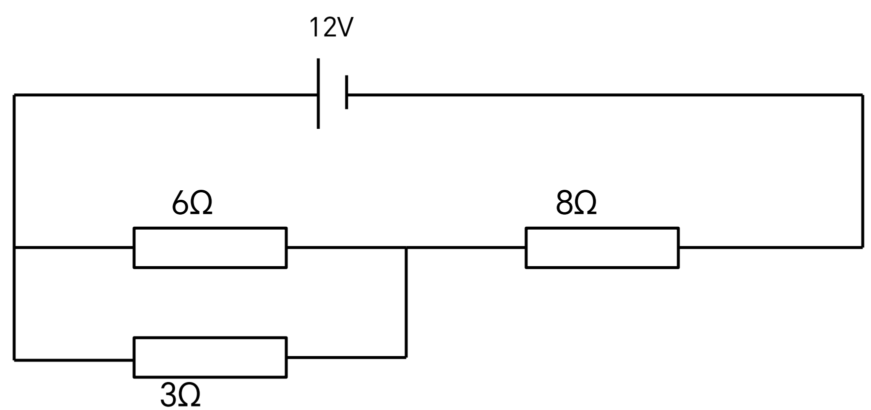


Figure 13.19: SAQ 13.1

Assignment

- **13.2.** A battery of e.m.f. 8 V and internal resistance *r* is connected to a 10 Ω resistor. Given that the terminal p.d. across the resistor is 5 V, calculate the value of the internal resistance of the battery.

- **13.3.** A battery *X* of e.m.f. of 6 V and internal resistance 2 Ω is connected in series with a battery *Y* of e.m.f. 4 V and internal resistance 8 Ω so that the two e.m.f.s act to oppose each other, calculate the terminal p.d. of *X* and *Y*.

- **13.4.** A light bulb has a resistance of 60 Ω when lit. How much current will flow through it when it is connected across 120 V, its normal operating voltage?

- **13.5.** A dry cell has an e.m.f. of 1.50 V. Its terminal potential drops to zero when a current of 10 A passes through it. What is its internal resistance?

- **13.6.** A metal rod is 2 m long and 8 mm in diameter. Calculate its resistivity if the resistance of the rod is 7×10^{-8} Ω.

- **13.7.** The constant resistance of a resistor is 30.0 Ω, independent of the temperature. The resistor is made up of an aluminum resistor with resistance R_1 at 0°C in series with a carbon resistor with resistance R_2 at 0°C. Evaluate R_1 and R_2, given that the temperature coefficient for aluminum and carbon are 3.9×10^{-3}°C^{-1} and -0.50×10^{-3}°C^{-1} respectively.

Assignment

- **13.8.** State Ohm's law and differentiate between Ohmic and Non-Ohmic conductors.
- **13.9.** A wire that has a resistance of 5.0 Ω is made into a new wire three times as long as the original. What is the new resistance?
- **13.10.** How many electrons per second pass through a section of a conductor carrying a current of 0.70 A?
- **13.11.** Calculate the resistance that will be connected in series with a cell of 1.54 V, 0.10 Ω, given that the terminal p.d. is 1.40 V.
- **13.12.** A generator supplies a current of 10 A to charge a storage battery which has an open-circuit voltage of 5.6 V. If the voltmeter connected across the generator reads 6.7 V, what is the internal resistance of the generator?
- **13.13.** Calculate the resistance of a 110 m long and 1.0 mm diameter aluminum wire of resistivity 2.8×10^{-8} Ωm.

Assignment

- **13.14.** Three 5 ohms resistors connected in parallel have a potential difference of 60V applied across the combination. The current in each resistor is A. 4A B. 36A C. 12A D. 24A
- **13.15.** How much current does a 60 W light bulb draw when connected to its proper voltage, 240 V?
- **13.16.** Calculate the current drawn from a heater having 1600 W at 240 V.
- **13.17.** Calculate the resistance of a 60 W/120 V bulb.
- **13.18.** Sparks of artificial 10.0 MV lightning had energy output of 0.125 MWs. How many coulombs of charge flowed?
- **13.19.** 90 C charge is transferred in one minute when a current *I* flows through a conductor whose terminals are connected across a potential difference of 100 V. Calculate the value of *I* and the power expended in heating the conductor if all the electrical energy is converted into heat.