ENGINEERING DRAWING

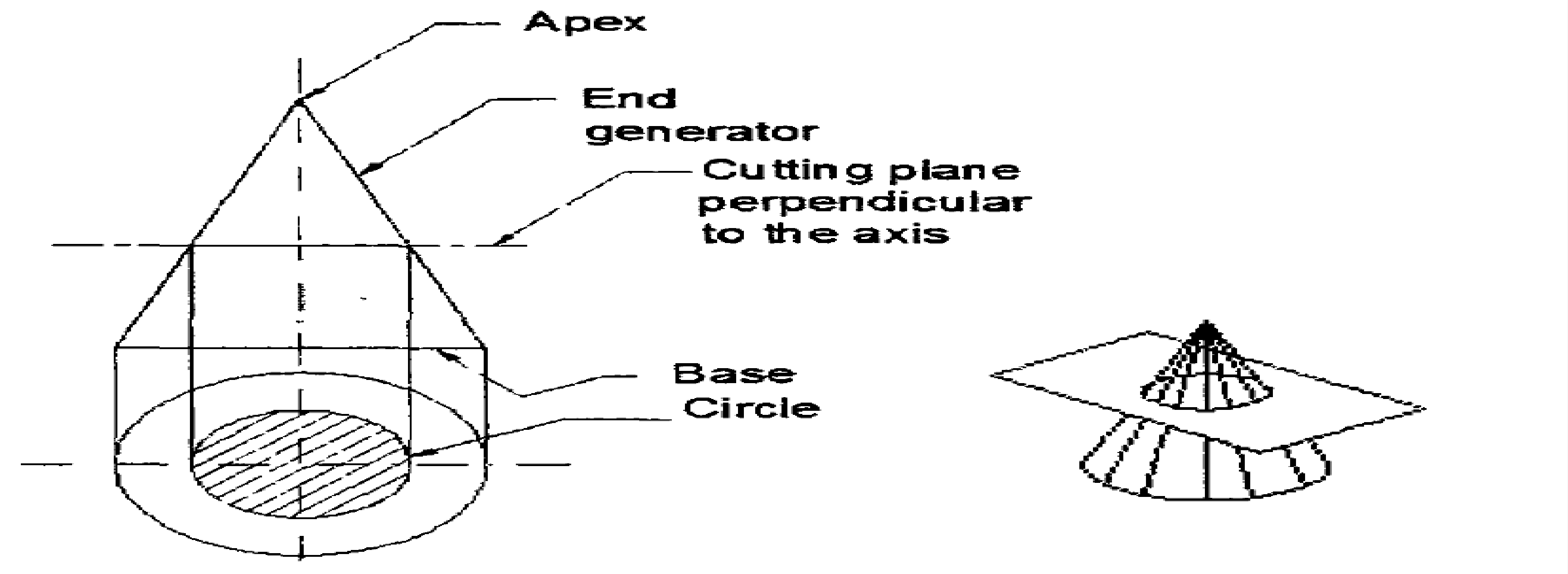
LECTURE 4

GEOMETRIC CONSTRUCTION:
CONIC SECTIONS

CONIC SECTIONS

Cone is formed when a right angled triangle with an apex and angled θ is rotated about its altitude as the axis. The length or height of the cone is equal to the altitude of the triangle and the radius of the base of the cone is equal to the base of the triangle. The apex angle of a cone is 2θ.

A conic section is a section cut by a plane passing through a cone. These sections are bounded by various kinds of shapes. Depending upon where the section is cut, the shape can be a triangle, a circle, an ellipse, a parabola or a hyperbola.



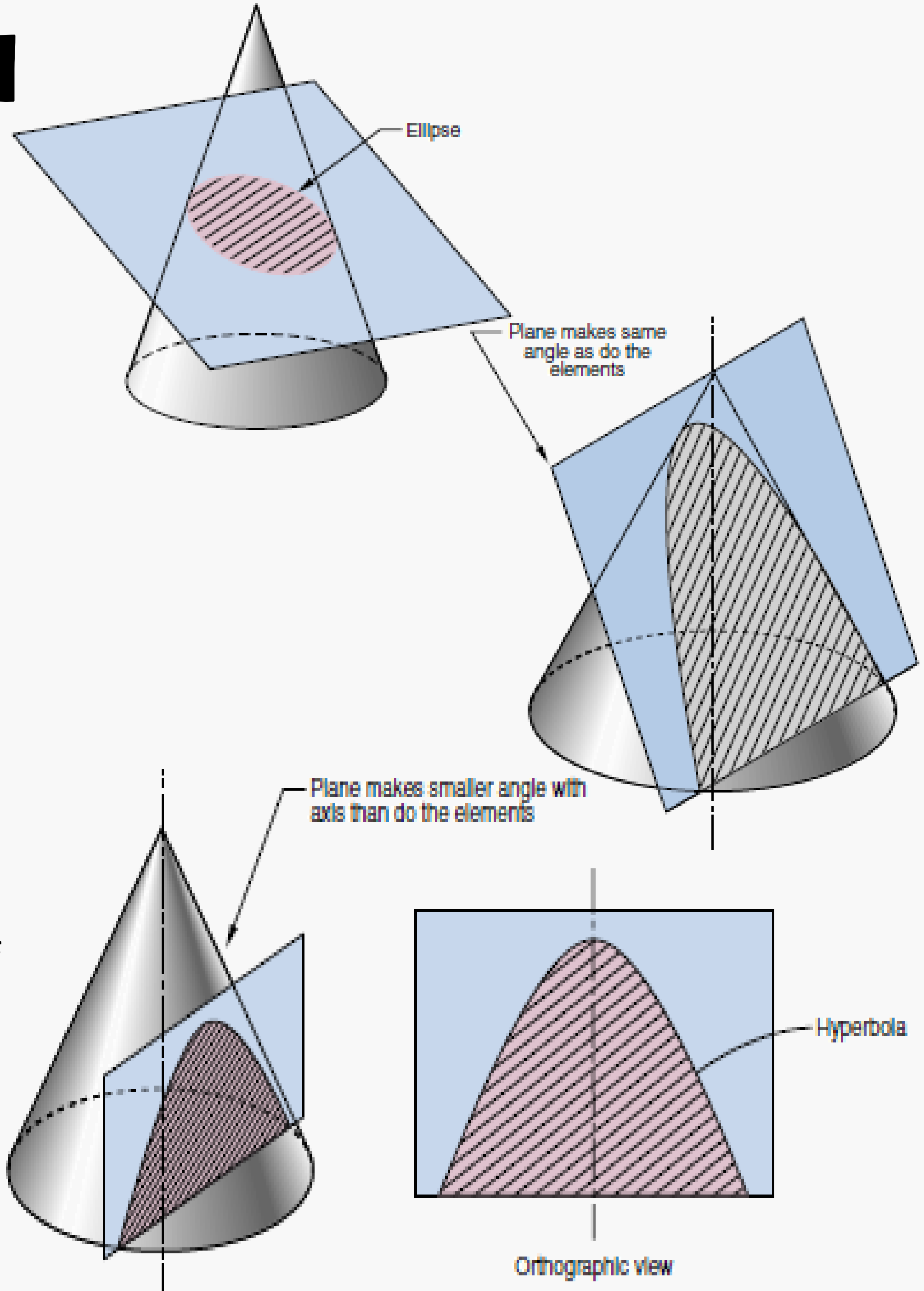
CONIC SECTION

Circle: This shape results when a plane passes through the cone, parallel with the base and perpendicular to the axis.

Ellipse: is a curve created when a plane passes through a right circular cone at an angle to the axis that is greater than the angle between the axis and the sides.

Parabola: is the curve created when a plane intersects a right circular cone parallel to the side (elements) of the cone.

Hyperbola: is the curve of intersection created when a plane intersects a right circular cone and makes a smaller angle with the axis than do the sides (elements).



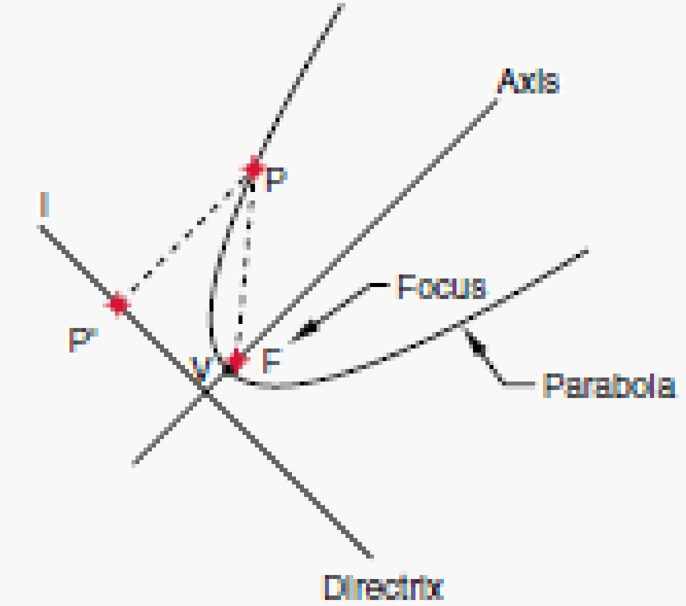
CONIC SECTIONAS LOCI OF A MOVING PART

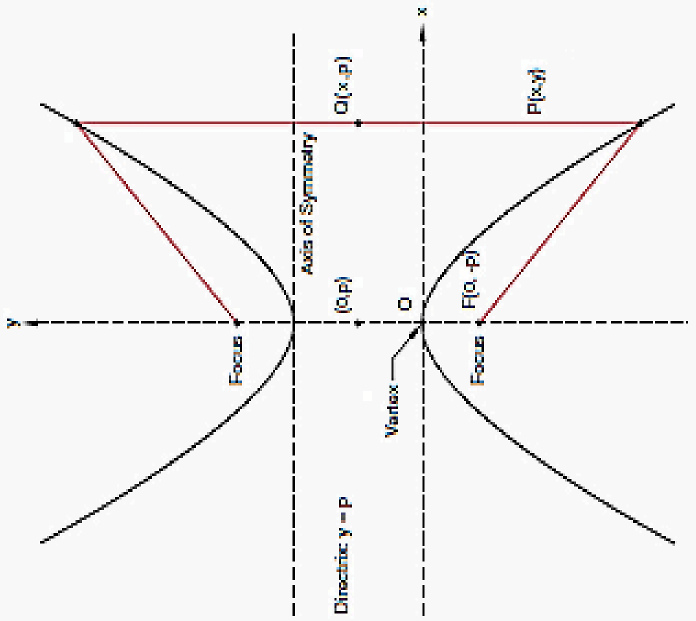
<u>Circle:</u> This is defined as the set of points that are equidistant from a given point called the centre.

<u>Ellipse:</u> This is defined a set of points in a plane whose sum of distances to two fixed point, called foci, is a constant.

<u>Parabola:</u> This is defined as the set of points in plane that are equidistant from a given point, called focus, and a given line, called a directrix.

<u>Hyperbola:</u> This is defined as the set of points in a plane whose distances from two fixed points, called the foci, in the plane have a constant difference.

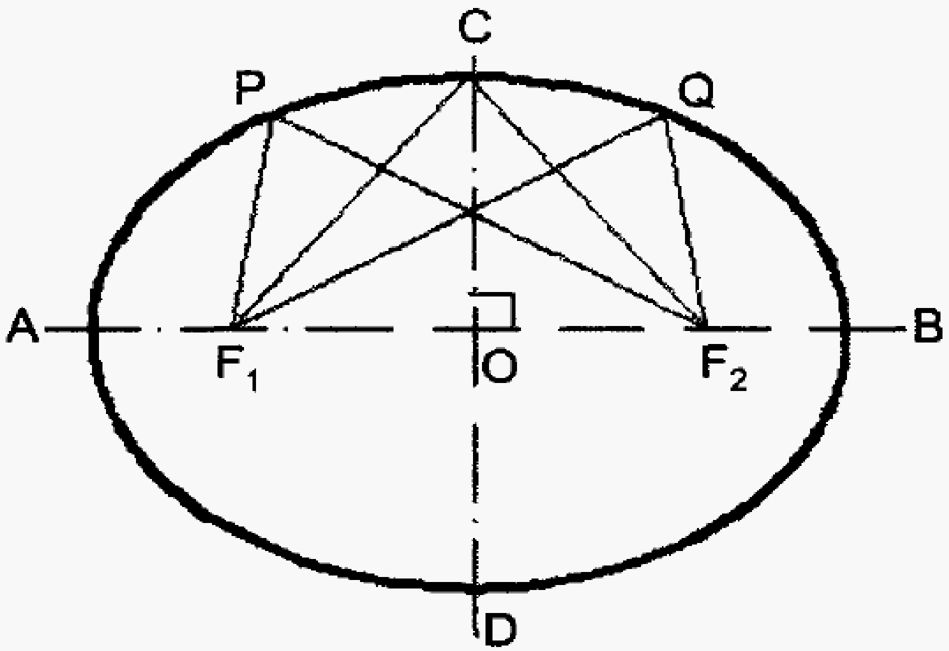




ELLIPSE

An ellipse is a curve traced by a point moving such that the sum of its distance from the two fixed points, foci, is constant and equal to the major axis.

Given the dimensions of major and minor axes, the ellipse can be drawn by (a) concentric circle method (b) Foci method (c) Trammel method (d) Oblong method



F_1 and F_2 are the two foci,
AB is the major axis
CD is the minor axis.

From the above definition:

$$\begin{aligned} PF_1 + PF_2 &= CF_1 + CF_2 \\ &= QF_1 + QF_2 \\ &= AB \end{aligned}$$

Also note that;

$$CF_1 = CF_2 = Half\ AB\ (Major\ axis)$$

ELLIPSE- Concentric Circle Method

STEPS

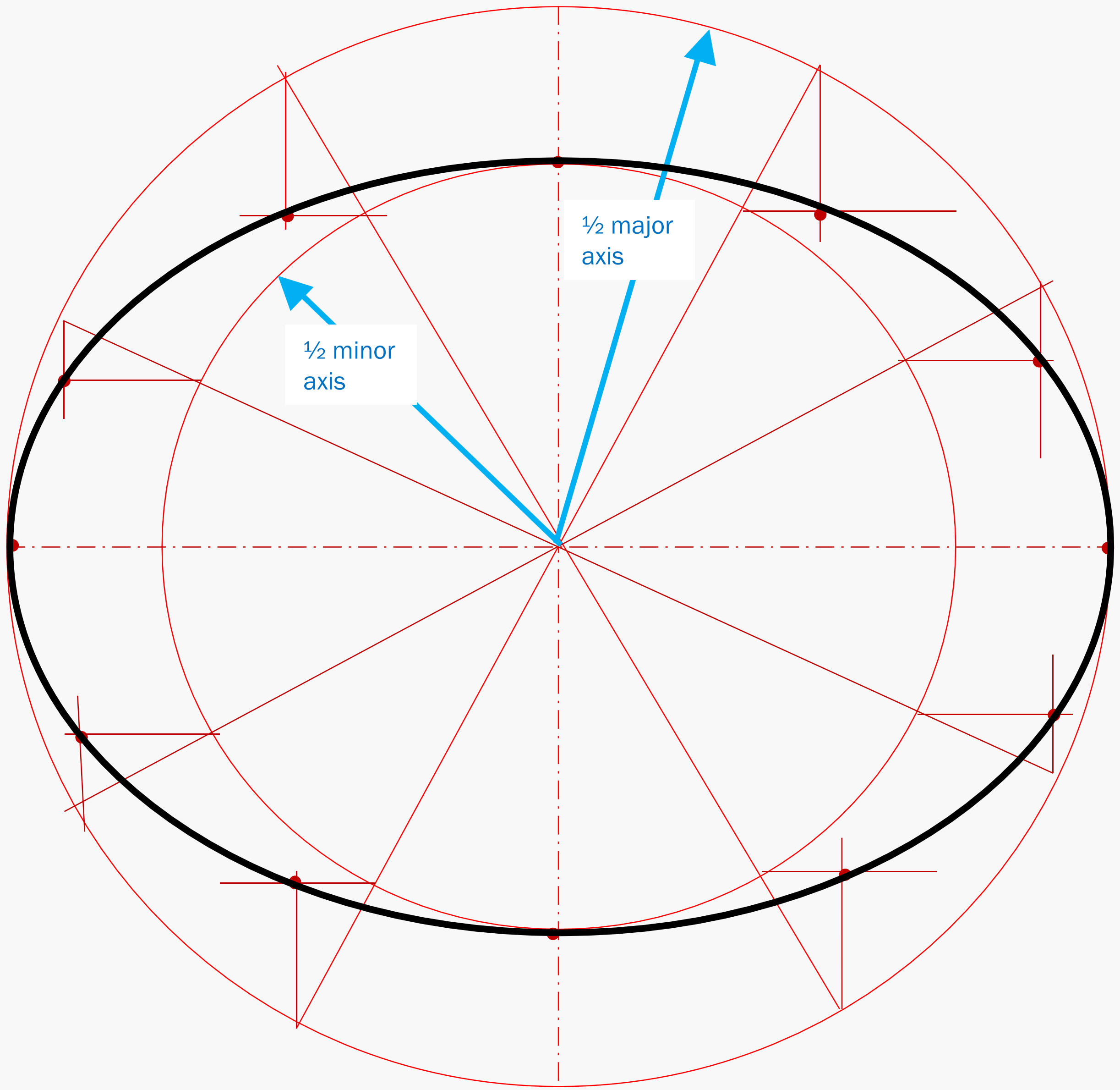
1. Given the major and minor diameters, construct two concentric circles with radii equal to ½ of major and ½ of minor axes.

2. Divide the circles into a convenient number of equal parts, say 12 (more divisions for large ellipses is advised).

3. Where radial lines intersect the inner circle, draw horizontal lines towards the larger circle.

4. Where the radial lines cross the larger circle, draw vertical lines to meet the horizontal lines.

5. Draw a neat curve through the intersections.



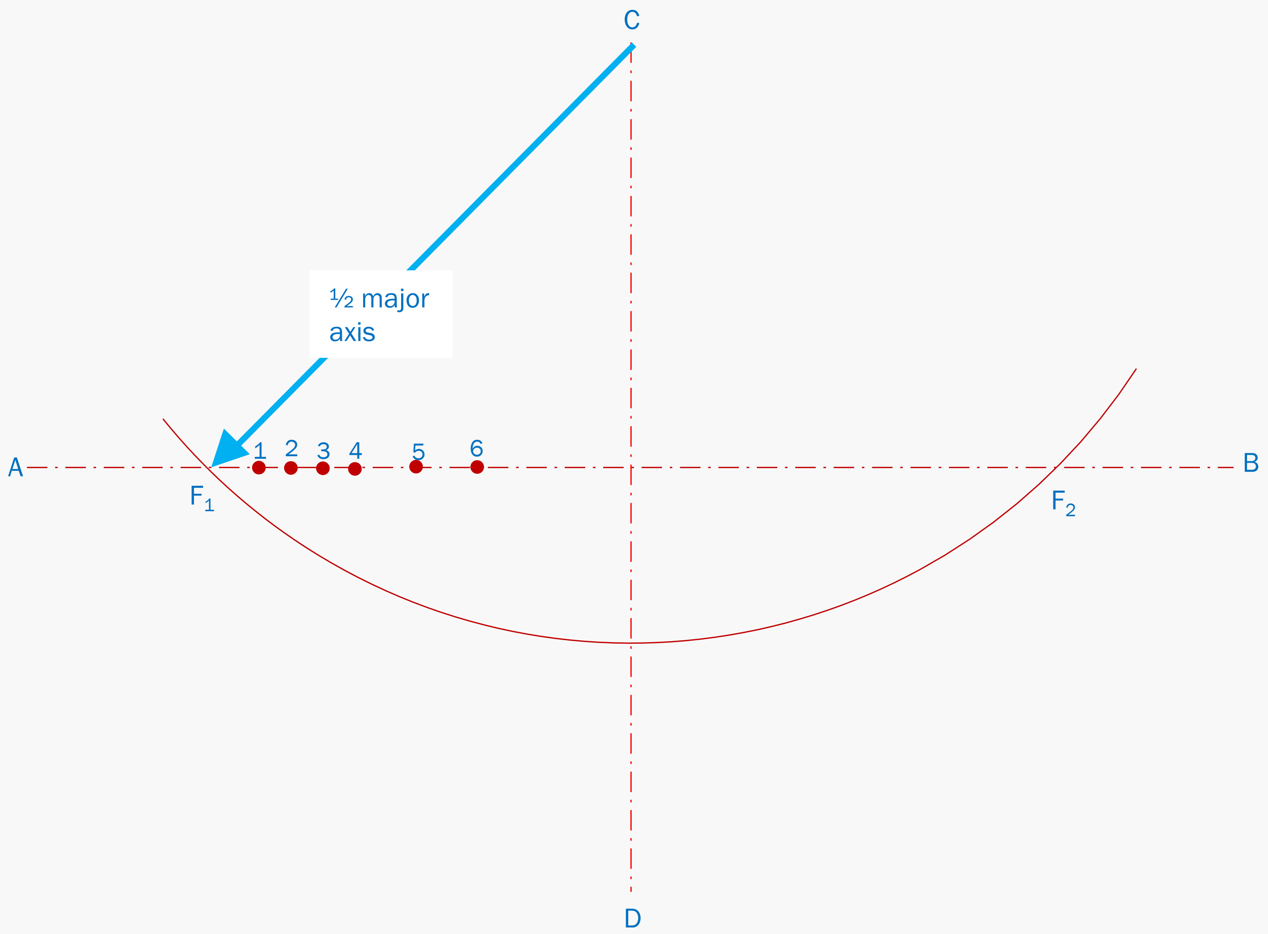
ELLIPSE- Foci Method

STEPS

1. Set out the major and minor axes, AB and CD respectively.

2. Find the focal points by drawing an arc from point C or D with radius OA (½ of major axis) to cut the major axis at F_1 and F_2.

3. Select a random array of points on the major axis between one focal points and the ellipse center (space those at the ends more closely). Number these in consecutive order.



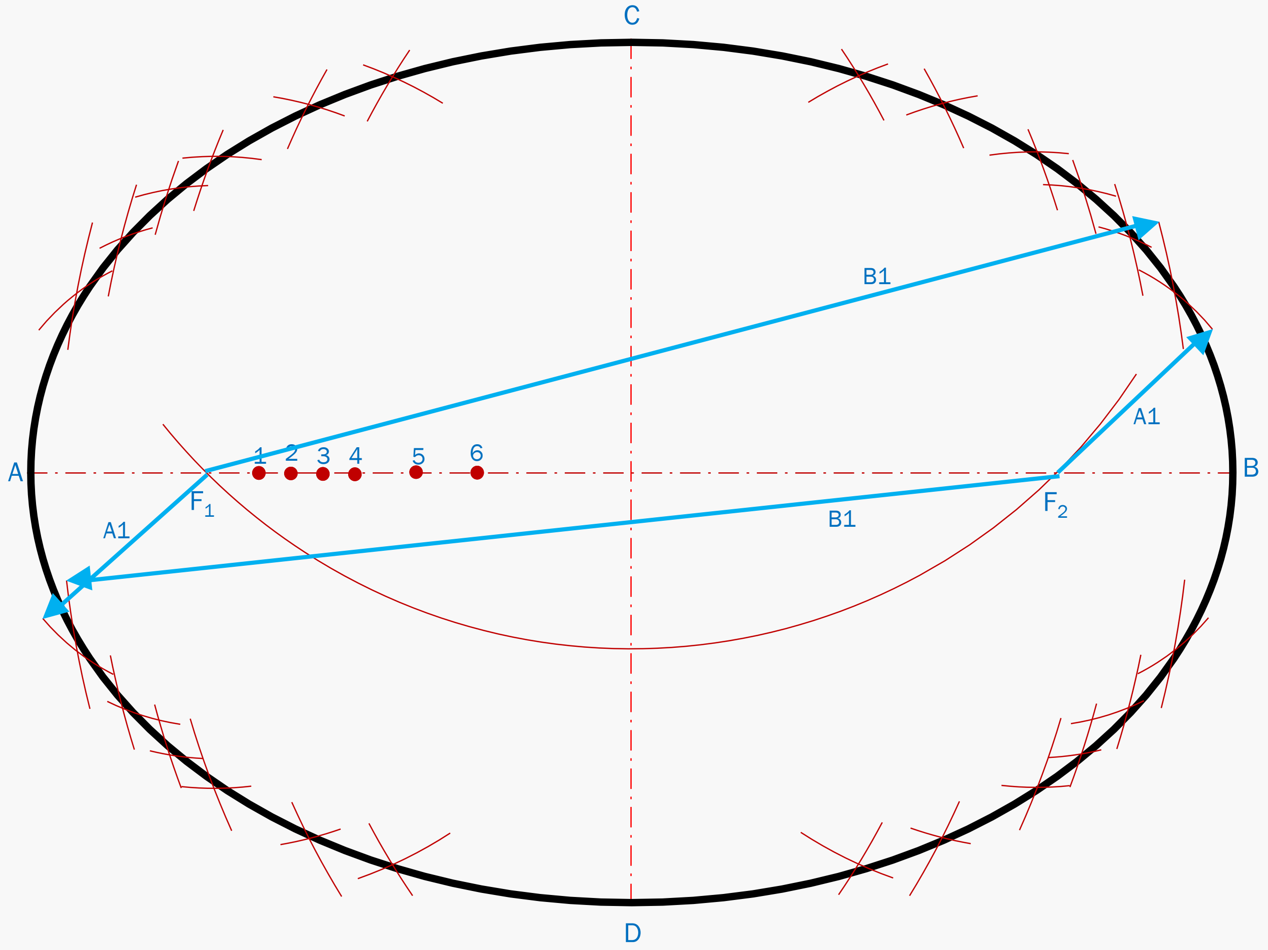
ELLIPSE- Foci Method

STEPS

4. Draw intersecting arcs whose sum of radii is AB from F_1 and F_2.

   A1,B1 from F_1 and F_2
   A2,B2 from F_1 and F_2
   etc

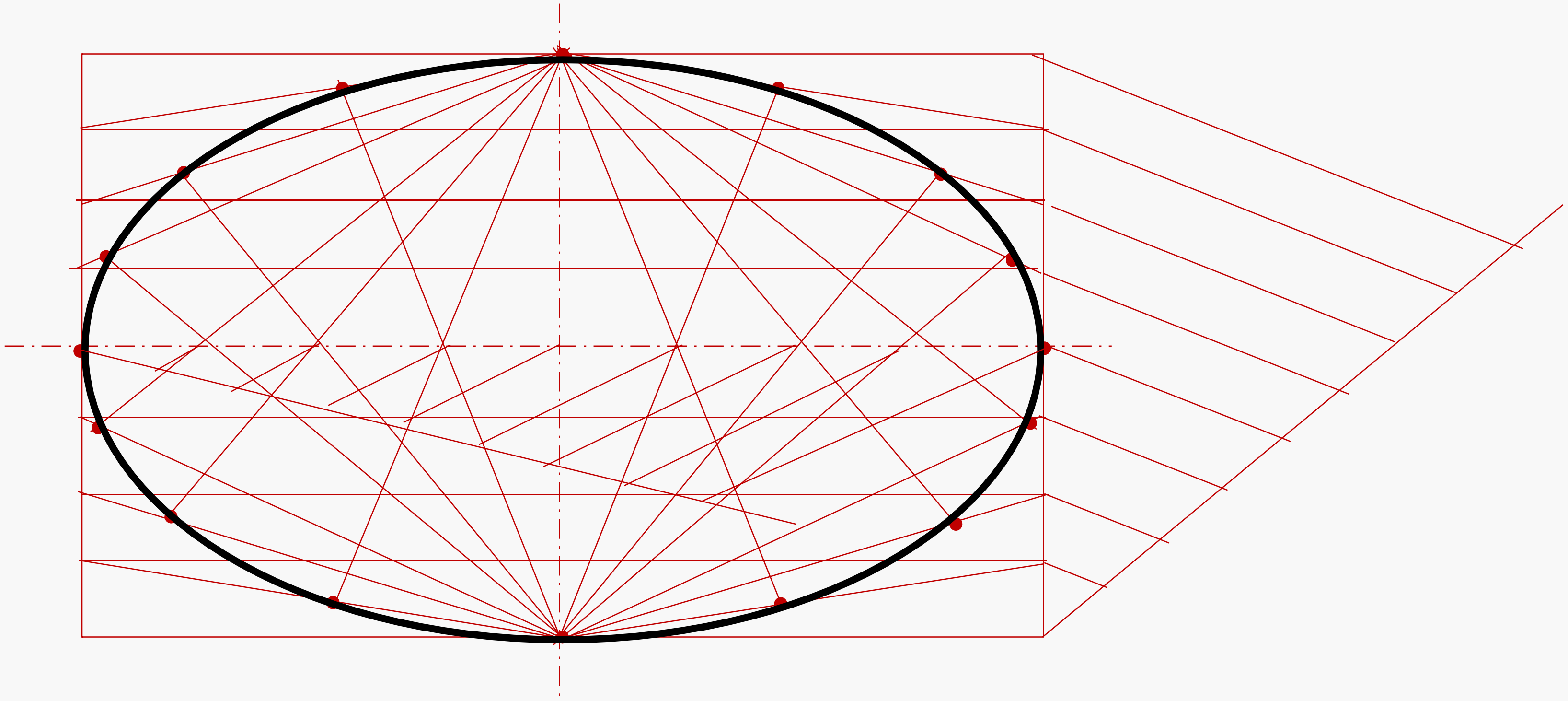
5. Draw a smooth curve through the points of intersection of the curves



ELLIPSE- Rectangle Method

STEPS

1. Draw a rectangle with length and breadth equal to the major and minor axes.
2. Divide the two shorter sides of the rectangle into the same number of equal parts.
3. Divide the major axis into the same number of equal parts.
4. From the points where the minor axis crosses the of the edge of the rectangle, draw intersecting lines as shown.
5. Draw a neat curve through the intersections.



PARABOLA – Rectangle Method

1. Draw a rectangle with dimensions as the rise (along the axis) and span (perpendicular to the axis).

2. Divide span into two equal halves.

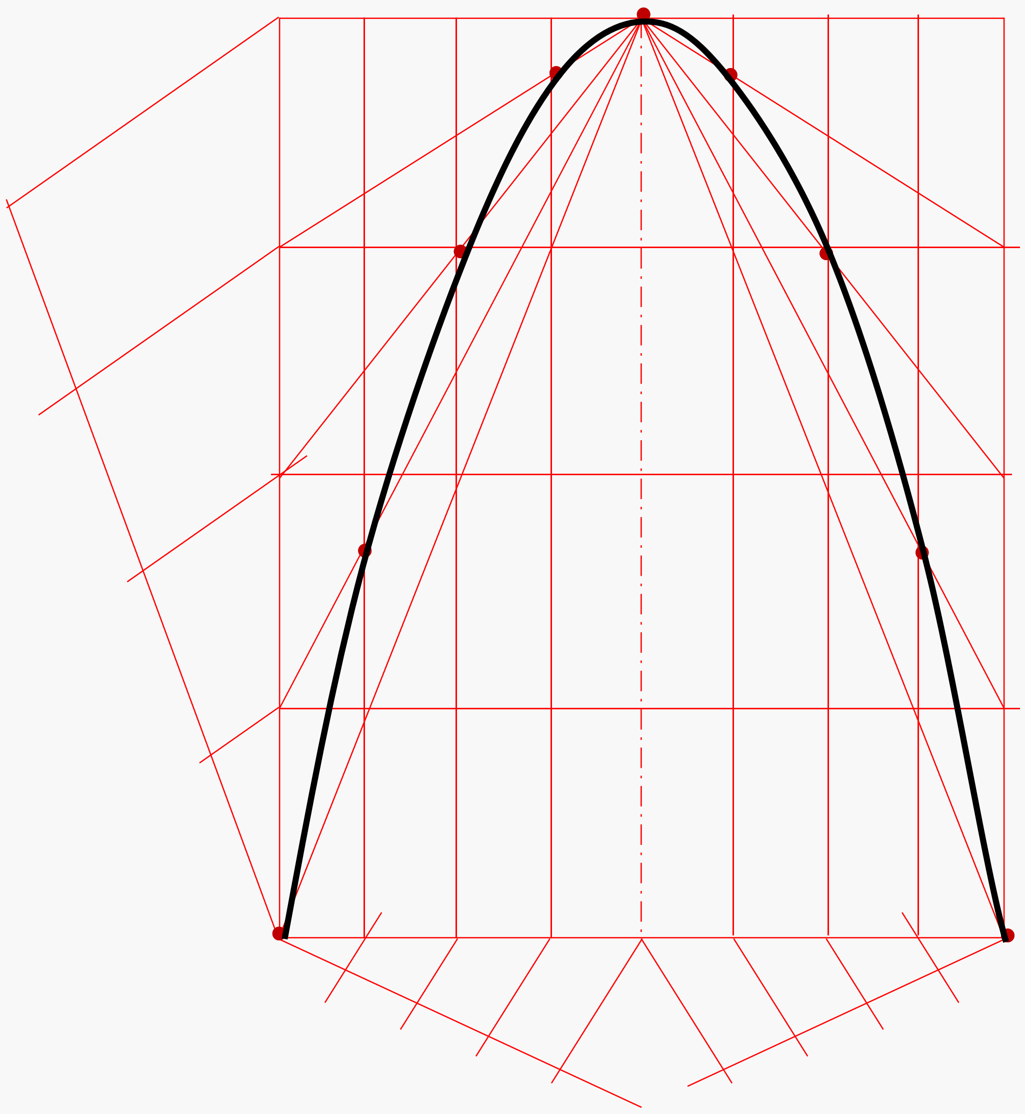
3. Divide half of the span into a number of parts, say x, such that the entire span is divided into 2x parts.

4. Divide the rise into the same number of x parts.

5. Draw lines from the mid-span to the edges of rectangle as follows.

6. Draw vertical lines to intersect the corresponding lines

7. Draw a smooth curve through the points of intersection.



PARABOLA – Offset Method

1. Given the sizes of the enclosing rectangle, distances AB (span) and AC (rise), construct a parallelogram.

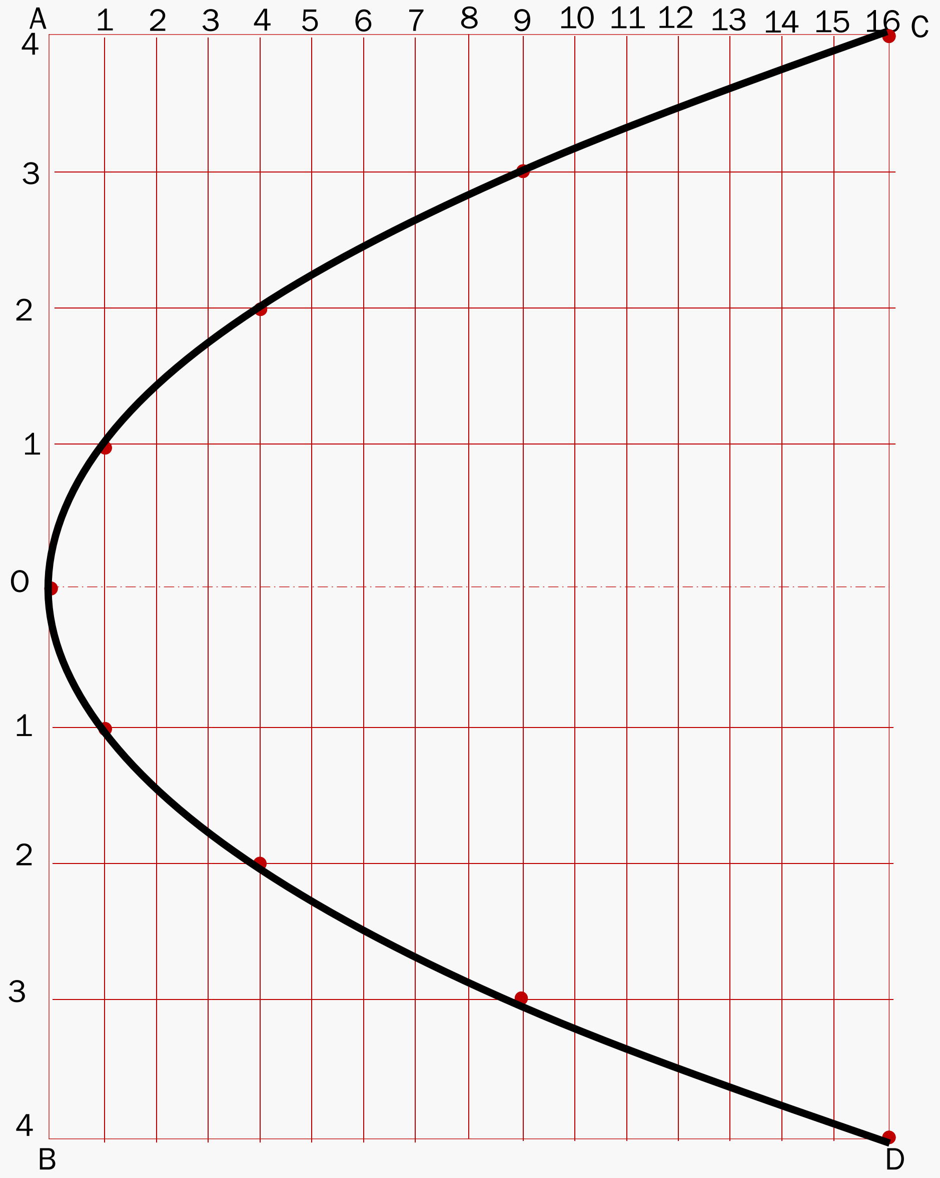
2. Bisect AC to locate point O.

3. Divide OA into four equal parts. (Similarly, divide OB into four equal parts).

4. The offsets vary in length as the square of their distances from O. Since OA is divided into four equal parts, distance AC will be divided into 4^2, or 16, equal divisions.

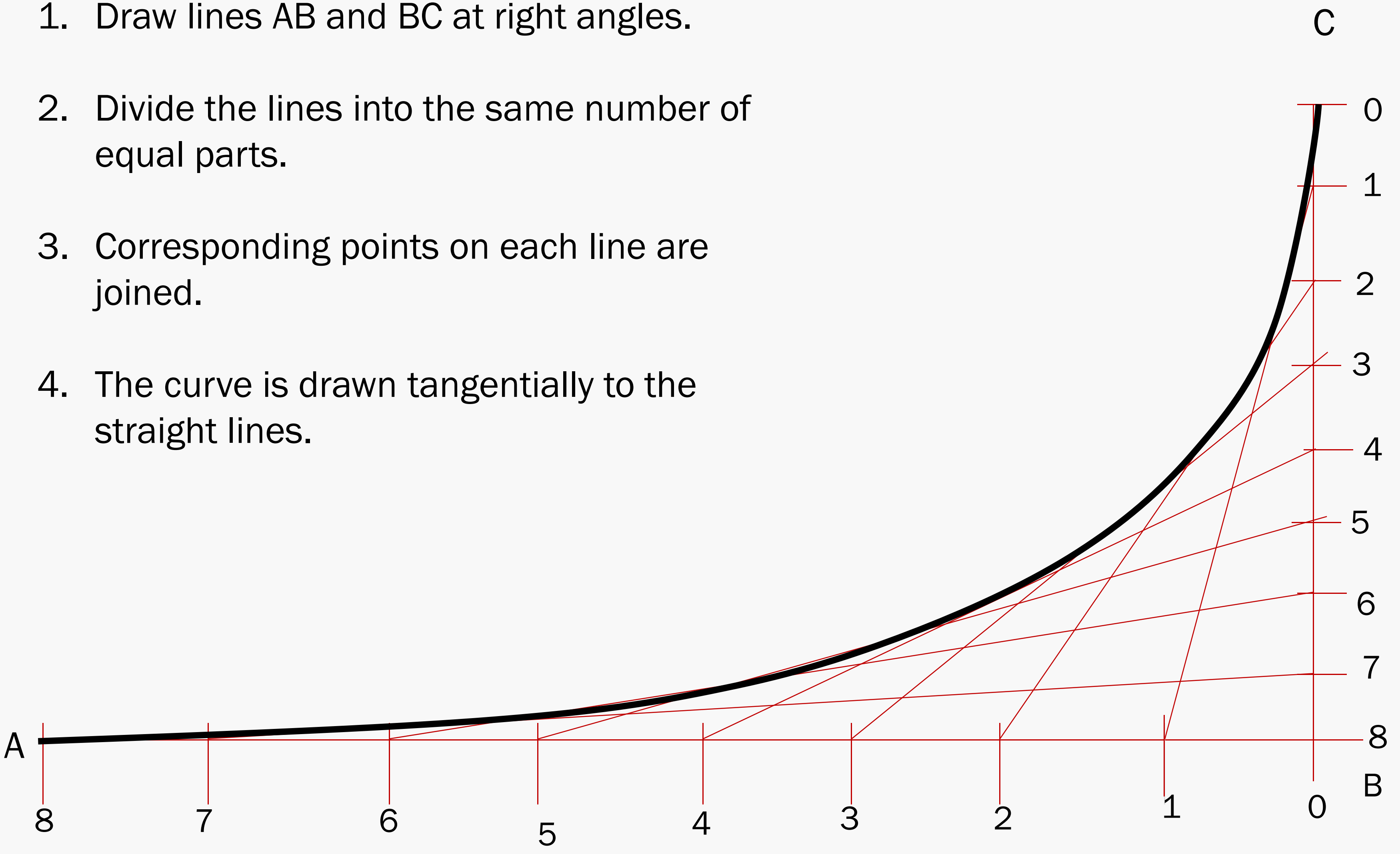
5. Locate points such that the point on the horizontal axis is the square of the value of point on the vertical axis.

6. Draw a smooth curve through the points



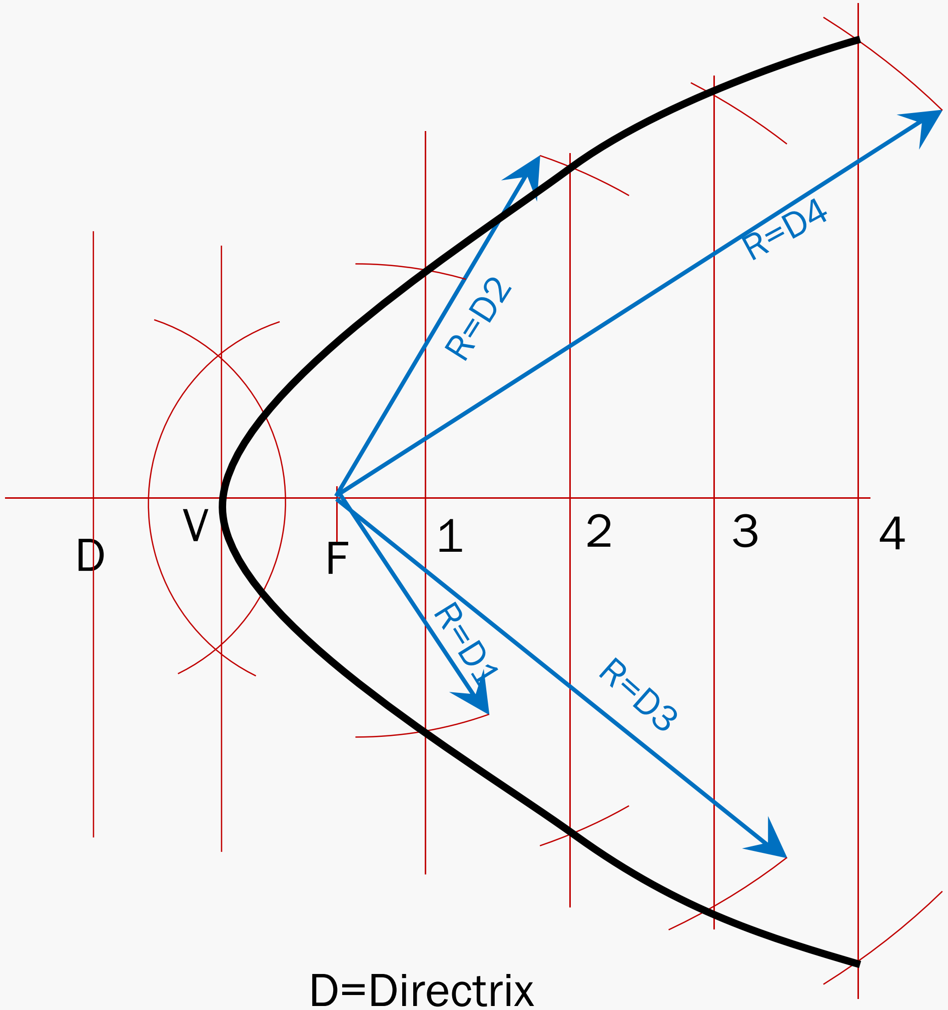
PARABOLA - Envelope Method

1. Draw lines AB and BC at right angles.
2. Divide the lines into the same number of equal parts.
3. Corresponding points on each line are joined.
4. The curve is drawn tangentially to the straight lines.



PARABOLA- Locus Method

1. Locate the focus, F, and directrix, D.
2. Locate the vertex by bisecting FD
3. Draw lines parallel to the directrix.
4. With F as centre, and radii equal to the distance of the directrix from the parallel lines, draw arcs to cut the parallel lines
5. Draw a smooth curve through the points of intersection.



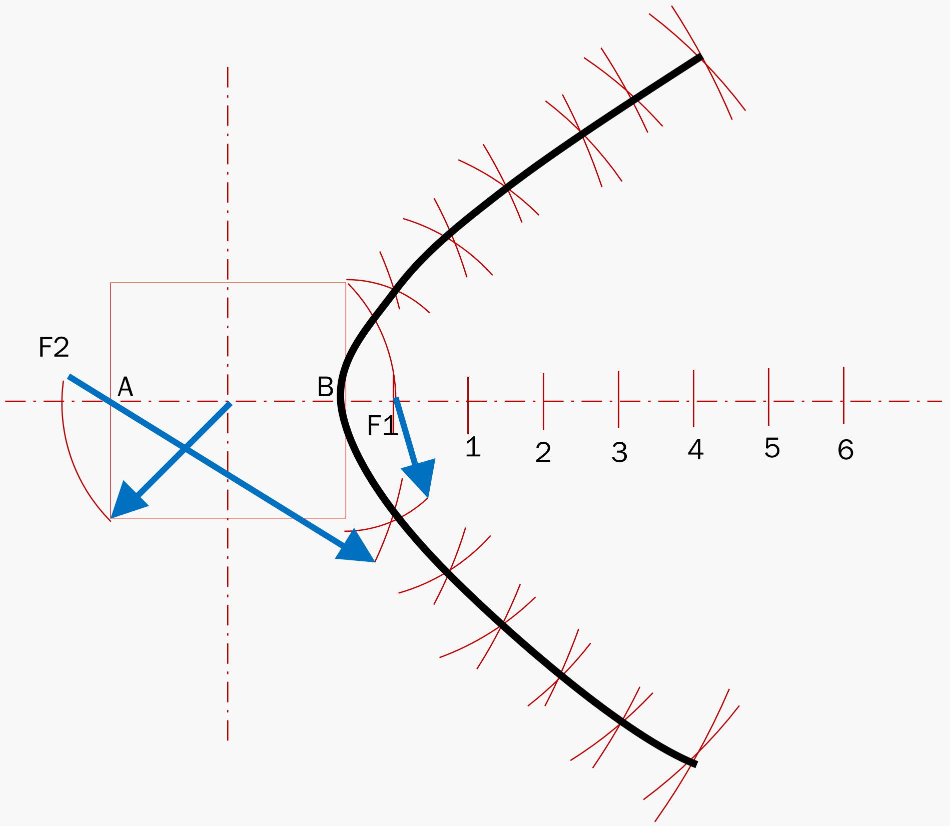
D=Directrix

F=Focus

V=Vertex

HYPERBOLA: Locus Method 1 (given transverse axis)

1. Draw a square whose sides equal the length of the transverse axis, AB.
2. Locate the centre of the transverse axis.
3. From the centre of the transverse axis, and radii equal to half the length of the diagonal, swing arcs to intersect the horizontal line through the centre of the square. This locates focus F1 and focus F2.
4. Progressing outward along the horizontal line, mark off equal spaces of arbitrary length from the focus points.
5. With A1 as radius, swing an arc from focus F1. With radius B1, swing an arc from focus F2.
6. Repeat the same procedure for as many points as required. A2,B2; A3,B3; etc
7. Using an irregular curve, carefully complete the hyperbola curve.



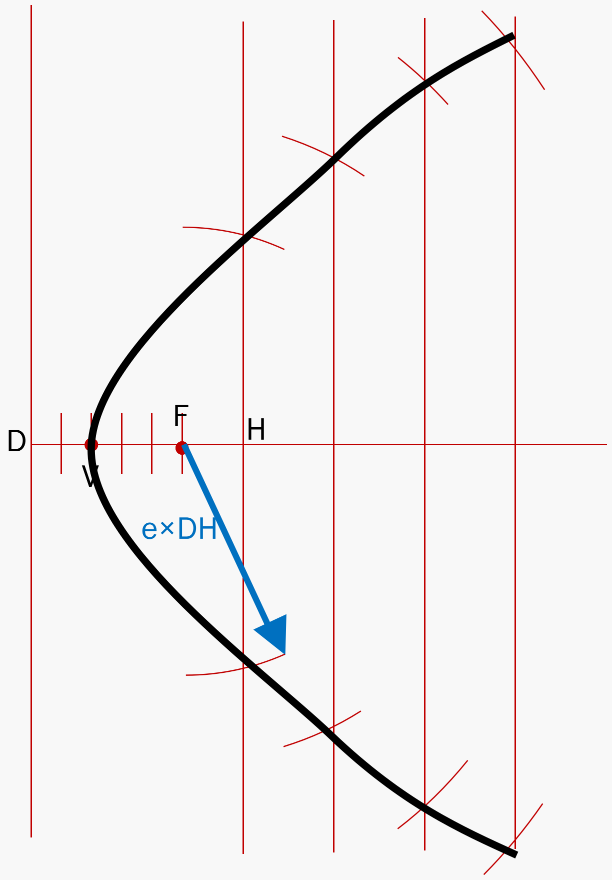
HYPERBOLA: Locus Method (Given eccentricity)

Eccentricity, e, is the ratio of the distances of a point to the focus and directrix. (For hyperbola, e>1; for parabola, e=1; for ellipse, e<1)

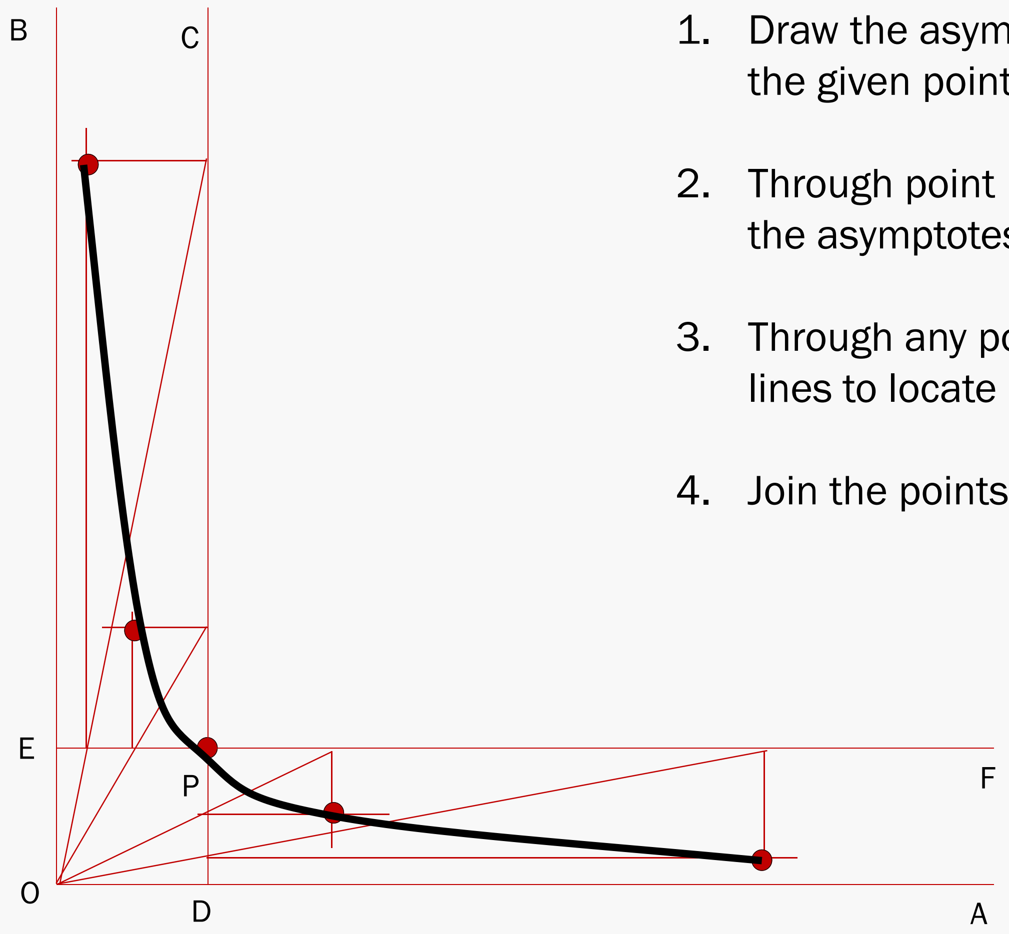
To draw a hyperbola with eccentrity of say 3/2,

1. Locate the focus, F, and directrix, D.
2. Divide DF into five (3+2) parts and locate the vertex 2 divisions from D.
3. Draw lines parallel to the directrix.
4. With F as centre, draw arcs to cut the parallel lines above and below the axis, taking the radius as:

$[(Distance\ between\ directrix\ and\ line)$



HYPERBOLA: Rectangular Method



Lines which are tangents to he hyperbola at infinity are called asymptotes. Given asymptotes and one point on the curve:

1. Draw the asymptotes OA and OB and locate the given point, P.

2. Through point P, draw CD and EF parallel to the asymptotes.

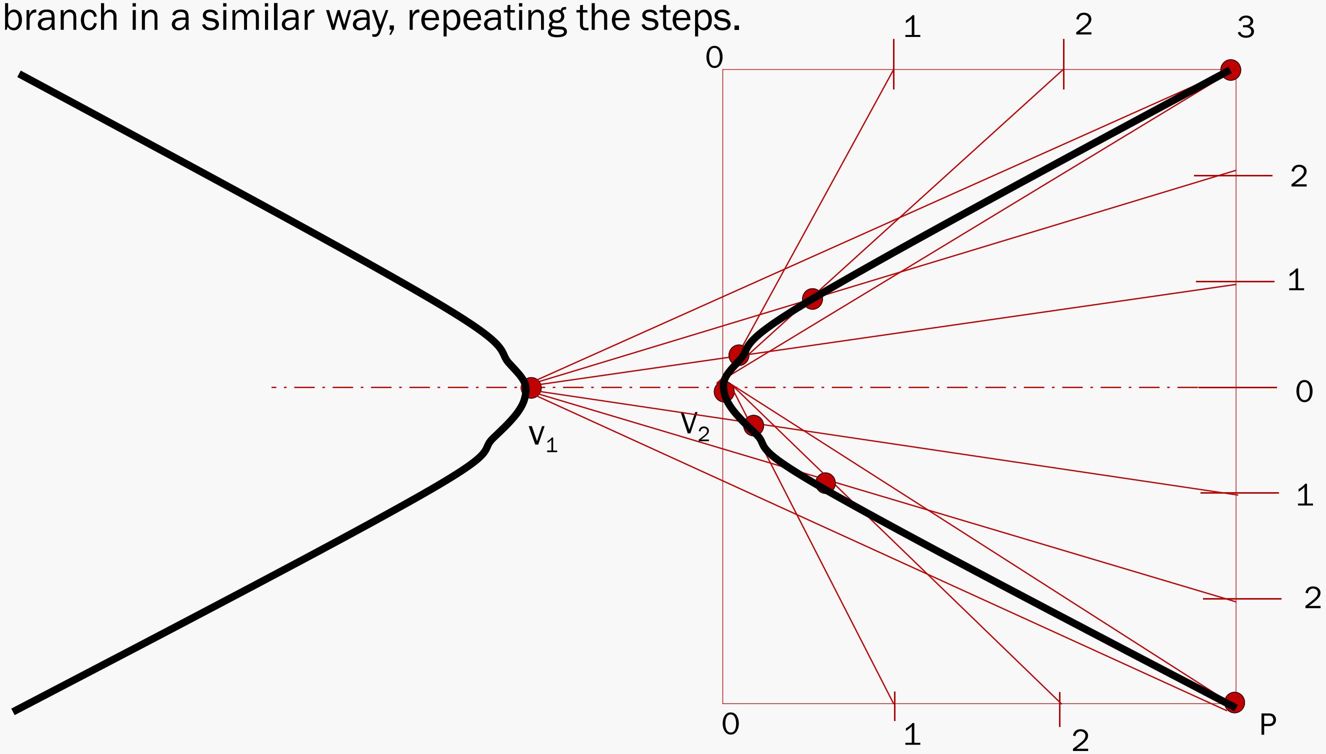
3. Through any points on CD and EF, draw lines to locate other points on the curve.

4. Join the points with a smooth curve.

HYPERBOLA: Circumscribing Rectangular Method

The hyperbola has two branches, each with a focus and directrix. Both branches have the same eccentricity.
Given both vertices, along with a point on one branch of the hyperbola,

1. Locate the vertices, V_1 and V_2 and given point P.
2. Construct a rectangle through P and its associated vertex.
3. Divide the rectangle into two by the axis of symmetry.
4. Divide the two sides of the rectangles into the same number of equal parts
5. Join the vertices to them as shown to locate the points on one branch of the hyperbola.
6. Draw the second branch in a similar way, repeating the steps.



END OF LECTURE 4