PHS 122 (ELECTRICITY AND MAGNETISM)

Coulomb's Law and Electric Fields

Coulomb's Law: Suppose that two point charges, $q_\bullet$ and $q'_\bullet$, are a distance r apart in vacuum. If $q_\bullet$ and $q'_\bullet$ have the same sign, the two charges repel each other; if they have opposite signs, they attract each other. The force experienced by either charge due to the other is called a **Coulomb** or **electric force** and it is given by **Coulomb's Law**

$$\text{[in vacuum]} \qquad F_E = k_0 \frac{q_\bullet q'_\bullet}{r^2} \qquad (24.1)$$

Textbooks vary somewhat on how they write Coulomb's Law, though the different statements are more or less equivalent. Here Eq. (24.1) is the traditional and most common statement, usually without the dots that remind us we are dealing with point charges or small charged spheres. Some books give the law as

$$F = k\,Q_1 Q_2 / r^2$$

Because charge can be positive or negative, the force can be positive (repulsive) or negative (attractive). To try to avoid sign confusion, some authors write the law as

$$F = k\,|q_1||q_2| / r^2$$

while still others give it as

$$F = k\,|q_1 q_2| / r^2$$

As always in the SI, distances are measured in meters, and forces in newtons. The SI unit for charge is the *coulomb* (C). The constant k_0

(corresponding to vacuum) in Coulomb's Law has the value

$$k_0 = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \tag{24.2}$$

which is often approximated as $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$. Often, k_0 is replaced by $1/4\pi\varepsilon_0$, where $\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$ is called the **permittivity of free space**. Then Coulomb's Law becomes

$$\text{[in vacuum]} \qquad F_E = \frac{1}{4\pi\varepsilon_0}\frac{q_* q_*'}{r^2} \tag{24.3}$$

When the surrounding medium is not a vacuum, forces caused by induced charges in the material reduce the force between point charges. If the material has a **dielectric constant** K, then ε_0 in Coulomb's Law must be replaced by $K\varepsilon_0 = \varepsilon$, where ε is called the *permittivity of the material*. Then

$$\text{[material medium]} \qquad F_E = \frac{1}{4\pi\varepsilon}\frac{q_* q_*'}{r^2} = \frac{k_0}{K}\frac{q_* q_*'}{r^2} \tag{24.4}$$

For vacuum, $K = 1$; for air, $K = 1.000\,6$. See Table 24-1.

TABLE 24-1
The Permittivity (ε) and Relative Permittivity* ($\varepsilon/\varepsilon_0$) of Some Common Substances

SUBSTANCE	PERMITTIVITY ($\text{C}^2/\text{N} \cdot \text{m}^2$)	RELATIVE PERMITTIVITY ($\varepsilon/\varepsilon_0$)
Vacuum	8.85×10^{-12}	1.00000
Air	8.85×10^{-12}	1.00054
Body tissue	71×10^{-12}	8
Glass	44×10^{-12}–89×10^{-12}	5–10
Mica	27×10^{-12}–53×10^{-12}	3–6
Nylon	31×10^{-12}	3.5
Paper	18×10^{-12}–35×10^{-12}	2–4
Polyethylene	20×10^{-12}	2.3
Polystyrene	23×10^{-12}	2.6
Rubber	18×10^{-12}–27×10^{-12}	2–3
Silicone oil	19×10^{-12}–25×10^{-12}	2.2–2.8
Sodium chloride	50×10^{-12}	5.6
Teflon	19×10^{-12}	2.1
Ethanol (25 °C)	2.2×10^{-10}	24.3
Methanol (20 °C)	3.0×10^{-10}	33.6
Water (20 °C)	7.1×10^{-10}	80

*Also called the *dielectric constant*.

Coulomb's Law also applies to charged conducting spheres and spherical shells, as well as to uniform spheres of charge. This is true provided that these are all small enough, in comparison to their separations, so that the charge distribution on each doesn't become asymmetrical when two or more of them interact. In that case, r, the distance between the centers of the spheres, must be much larger than the sum of the radii of the two spheres.

Charge Is Quantized: The magnitude of the smallest charge ever measured is denoted by e (called the **quantum of charge**), where $e = 1.602\ 18 \times 10^{-19}$ C. All free charges, ones that can be isolated and measured, are integer multiples of e. The electron has a charge of $-e$, while the proton's charge is $+e$. Although there is good reason to believe that quarks carry charges of magnitude $e/3$ and $2e/3$, they only exist in bound systems that have a net charge equal to an integer multiple of e.

Conservation of Charge: The algebraic sum of the charges in the universe is constant. When a particle with charge $+e$ is created, a particle with charge $-e$ is simultaneously created in the immediate vicinity. When a particle with charge $+e$ disappears, a particle with charge $-e$ also disappears in the immediate vicinity. Hence, the net charge of the universe remains constant.

The Test-Charge Concept: A **test-charge** is a very small charge that can be used in making measurements on an electric system. It is assumed that such a charge, which is tiny both in magnitude and physical size, has a negligible effect on its environment.

An Electric Field is said to exist at any point in space when a test charge, placed at that point, experiences an electrical force. The direction of the electric field at a point is the same as the direction of the force experienced by a *positive* test charge placed at the point.

Electric field lines can be used to sketch electric fields. The line through a point has the same direction at that point as the electric field. Where the field lines are closest together, the electric field is largest. Field lines come out of positive charges (because a positive charge repels a positive test charge) and come into negative charges (because they attract the positive test charge).

The Strength of the Electric Field ($\vec{\mathbf{E}}$) at a point is equal to the force experienced by a unit positive test charge placed at that point. Because the electric field strength is a force per unit charge, it is a vector quantity. The units of $\vec{\mathbf{E}}$ are N/C or (see Chapter 25) V/m.

If a charge q is placed at a point where the electric field due to other charges is $\vec{\mathbf{E}}$, the charge will experience a force $\vec{\mathbf{F}}_E$ given by

$$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}} \tag{24.5}$$

If q is negative, $\vec{\mathbf{F}}_E$ will be opposite in direction to $\vec{\mathbf{E}}$.

Electric Field Due to a Point Charge: To find E (the signed magnitude of $\vec{\mathbf{E}}$) due to a point charge $q_{\bullet}$, we make use of Coulomb's Law. If a point charge $q'_{\bullet}$ is placed at a distance r from the charge $q_{\bullet}$, it will experience a force

$$F_E = \frac{1}{4\pi\varepsilon}\frac{q_{\bullet}q'_{\bullet}}{r^2} = q'_{\bullet}\left(\frac{1}{4\pi\varepsilon}\frac{q_{\bullet}}{r^2}\right)$$

But if a point charge $q'_{\bullet}$ is placed at a position where the electric field is E, then the force on $q'_{\bullet}$ is

$$F_E = q'_{\bullet} E \tag{24.6}$$

Comparing these two expressions for F_E, we see that the ***electric field of a point charge*** $q_{\bullet}$ is

[material medium] $$E_{\bullet} = \frac{1}{4\pi\varepsilon}\frac{q_{\bullet}}{r^2} \tag{24.7}$$

The same relation applies at points outside of a small spherical charge q. For q positive, E is positive and $\vec{\mathbf{E}}$ is directed radially outward from q; for q negative, E is negative and $\vec{\mathbf{E}}$ is directed radially inward.

Superposition Principle: The force experienced by a charge due to other charges is the vector sum of the Coulomb forces acting on it due to these other charges. Similarly, the electric intensity $\vec{\mathbf{E}}$ at a point due to several charges is the vector sum of the intensities due to the individual charges.

SOLVED PROBLEMS

24.1 [I] Two small spheres in vacuum are 1.5 m apart center-to-center. They carry identical charges. Approximately how large is the charge on each if each sphere experiences a force of 2 N?

The diameters of the spheres are small compared to the 1.5 m separation. We may therefore approximate them as point charges. Coulomb's Law, $F_E = k_0 q_{\bullet 1} q_{\bullet 2}/r^2$, leads to

$$q_{\bullet 1} q_{\bullet 2} = q^2 = \frac{F_E r^2}{k_0} = \frac{(2\ \text{N})(1.5\ \text{m})^2}{9\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 5\times 10^{-10}\ \text{C}^2$$

from which $q = 2 \times 10^{-5}$ C.

24.2 [I] Repeat Problem 24.1 if the spheres are separated by a center-to-center distance of 1.5 m in a large vat of water. The dielectric constant of water is about 80.

From Coulomb's Law,

$$F_E = \frac{k_0}{K}\frac{q^2}{r^2}$$

where K, the dielectric constant, is now 80. Then

$$q = \sqrt{\frac{F_E r^2 K}{k_0}} = \sqrt{\frac{(2\ \text{N})(1.5\ \text{m})^2(80)}{9\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 2\times 10^{-4}\,\text{C}$$

24.3 [I] A helium nucleus has a charge of $+2e$, and a neon nucleus has a charge of $+10e$, where e is the quantum of charge, 1.60×10^{-19} C. Find the repulsive force exerted on one by the other when they are separated by a distance of 3.0 nanometers (1 nm = 10^{-19} m). Assume the system to be in vacuum.

Nuclei have radii of order 10^{-15} m. We can assume them to be point charges in this case. Then

$$F_E = k_0\frac{q_\bullet q_\bullet'}{r^2} = (9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2)(10)(1.6\times 10^{-9}\ \text{C})^2}{(3.0\times 10^{-9}\ \text{m})^2} = 5\times 10^{-10}\ \text{N} = 0.51\ \text{nN}$$

24.4 [II] In the Bohr model of the hydrogen atom, an electron ($q = -e$) circles a proton ($q' = e$) in an orbit of radius 5.3×10^{-11} m. The attraction between the proton and electron furnishes the centripetal force needed to hold the electron in orbit. Find (*a*) the force of electrical attraction between the particles and (*b*) the electron's speed. The electron mass is 9.1×10^{-31} kg.

The electron and proton are essentially point charges. Accordingly,

$$(a)\quad F_E = k_0 \frac{q_\bullet q_\bullet'}{r^2} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(1.6 \times 10^{-19}\ \mathrm{C})^2}{(5.3 \times 10^{-11}\ \mathrm{m})^2} = 8.2 \times 10^{-8}\ \mathrm{N} = 82\ \mathrm{nN}$$

(*b*) The force found in (*a*) is the centripetal force, $m\upsilon^2/r$. Therefore,

$$8.2 \times 10^{-8}\ \mathrm{N} = \frac{m\upsilon^2}{r}$$

from which it follows that

$$\upsilon = \sqrt{\frac{(8.2 \times 10^{-8}\ \mathrm{N})(r)}{m}} = \sqrt{\frac{(8.2 \times 10^{-8}\ \mathrm{N})(5.3 \times 10^{-11}\ \mathrm{m})}{9.1 \times 10^{-31}\ \mathrm{kg}}} = 2.2 \times 10^6\ \mathrm{m/s}$$

24.10 [II] Compute (*a*) the electric field E in air at a distance of 30 cm from a point charge $q_{\bullet 1} = 5.0 \times 10^{-9}$ C, (*b*) the force on a charge $q_{\bullet 2} = 4.0 \times 10^{-10}$ C placed 30 cm from $q_{\bullet 1}$, and (*c*) the force on a charge $q_{\bullet 3} = -4.0 \times 10^{-10}$ C placed 30 cm from $q_{\bullet 1}$ (in the absence of $q_{\bullet 2}$).

$$(a)\quad E = k_0 \frac{q_{\bullet 1}}{r^2} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{5.0 \times 10^{-9}\ \mathrm{C}}{(0.30\ \mathrm{m})^2} = 0.50\ \mathrm{kN/C}$$

directed away from $q_{\bullet 1}$.

(*b*) $F_E = E_{q\bullet 2} = (500\ \mathrm{N/C})(-4.0 \times 10^{-10}\ \mathrm{C}) = 2.0 \times \mathrm{N} = 0.20\ \mu N$
directed away from $q_{\bullet 1}$.

(*c*) $F_E = E_{q\bullet 3} = (500\ \mathrm{N/C})(-4.0 \times 10^{-10}\ \mathrm{C}) = -0.20\ \mu N$
This force is directed toward $q_{\bullet 1}$.

24.12 [II] Three charges are placed on three corners of a square, as shown in Fig. 24-6. Each side of the square is 30.0 cm and the arrangement is in air. Compute $\vec{\mathbf{E}}$ at the fourth corner. What would be the force on a 6.00 μC charge placed at the vacant corner?

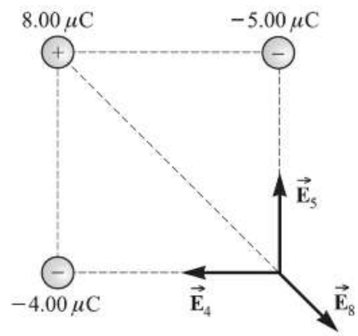


Fig. 24-6

The contributions of the three charges to the field at the vacant corner are as indicated. Notice in particular their directions, which correspond to the directions of the forces that would exist on a positive test charge if it was at that location. Their magnitudes are given by $E = k_0 q/r^2$ to be

$E_4 = 4.00 \times 10^5$ N/C $E_8 = 4.00 \times 10^5$ N/C $E_5 = 5.00 \times 10^5$ N/C

Because the E_8 vector makes an angle of 45.0° to the horizontal,

$$E_x = E_8 \cos 45.0° - E_4 = -1.17 \times 10^5 \text{ N/C}$$

$$E_y = E_5 - E_8 \cos 45.0° = 2.17 \times 10^5 \text{ N/C}$$

Using $E = \sqrt{E_x^2 + E_y^2}$ and $\tan\theta = E_y/E_x$, we find $E = 2.47 \times 10^5$ N at 118°.

The force on a charge placed at the vacant corner would be simply $F_E = Eq$. Since $q = 6.00 \times 10^{-6}$ C, we have $F_E = 1.48$ N at an angle of 118°.

SUPPLEMENTARY PROBLEMS

24.18 **[I]** Imagine two separated tiny interacting uniformly charged spheres. What happens to the electrostatic force on each of them if the charge on both is doubled and their separation is also doubled?

24.19 **[I]** What is the electrostatic force acting on each of two tiny uniformly charged spheres in vacuum if they both carry 1.00 C of charge and they are separated, center to center, by 1.00 m?

24.20 **[I]** What should be the separation in vacuum between two tiny spheres uniformly carrying charges of 10.0 nC and 20.0 nC if the force they exert on each other is to be 10.0 N?

24.21 **[I]** Compute the force on each of two electrons when they are separated in vacuum by a distance corresponding to the approximate size of an atom (0.100 nm).

24.22 **[I]** Determine the force that would exist between two uranium nuclei separated in vacuum by the approximate size of an atom (0.100 nm).

24.23 **[I]** Two very small charges, each of $-100\ \mu$C, are separated by 1.00 mm in ethanol at 25 °C. Determine the forces acting on each charge. [*Hint*: Use Table 24-1.]

24.24 **[I]** How many electrons are contained in 1.0 C of charge? What is the mass of the electrons in 1.0 C of charge?

24.25 **[I]** If two equal point charges, each of 1 C, were separated in air by a distance of 1 km, what would be the force between them?

24.26 **[I]** Determine the force between two free electrons spaced 1.0 angstrom (10^{-10} m) apart in vacuum.

24.27 **[I]** What is the force of repulsion between two argon nuclei that are separated in vacuum by 1.0 nm (10^{-9} m)? The charge on an argon nucleus is $+18e$.

24.29 **[II]** Three point charges are placed at the following locations on the x-axis: $+2.0\ \mu$C at $x = 0$, $-3.0\ \mu$C at $x = 40$ cm, $-5.0\ \mu$C at $x = 120$ cm. Find the force (*a*) on the $-3.0\ \mu$C charge, (*b*) on the $-5.0\ \mu$C charge.

24.30 **[II]** Four equal point charges of $+3.0\ \mu$C are placed in air at the four corners of a square that is 40 cm on a side. Find the force on any one of the charges.

24.31 **[II]** Four equal-magnitude point charges ($3.0\ \mu$C) are placed in air at the corners of a square that is 40 cm on a side. Two, diagonally opposite each other, are positive, and the other two are negative. Find the force on either negative charge.

24.32 **[II]** Charges of $+2.0$, $+3.0$, and $-8.0\ \mu$C are placed in air at the vertices of an equilateral triangle of side 10 cm. Calculate the magnitude of the force acting on the $-8.0\ \mu$C charge due to the other two charges.

24.33 **[II]** One charge of ($+5.0\ \mu$C) is placed in air at exactly $x = 0$, and a second charge ($+7.0\ \mu$C) at $x = 100$ cm. Where can a third be placed so as to experience zero net force due to the other two?

24.34 **[II]** Two identical tiny metal balls carry charges of $+3$ nC and -12 nC. They are 3 m apart in vacuum. (*a*) Compute the force of attraction. (*b*) The balls are now touched together and then separated to 3 cm. Describe the forces on them now.

24.35 **[II]** A charge of $+6.0\ \mu$C experiences a force of 2.0 mN in the $+x$-direction at a certain point in space. (*a*) What was the electric field at that point before the charge was placed there? (*b*) Describe the force a $-2.0\ \mu$C charge would experience if it were used instead of the $+6.0\ \mu$C charge.

Electric Potential; Capacitance

The Potential Difference between point-A and point-B is the work done against electrical forces in carrying a *unit* positive test-charge from A to B. We represent the potential difference between A and B by $V_B - V_A$ or just by V when there is no ambiguity. Its units are those of work per charge (joules/coulomb) and are designated as **volts** (V):

$$1\ \mathrm{V} = 1\ \mathrm{J/C}$$

Because work is a scalar quantity, so too is potential difference. Like work, potential difference may be positive or negative. The work W done in transporting a charge q from one point-A to a second point-B is

$$W = q(V_B - V_A) = qV \tag{25.1}$$

where the appropriate sign (+ or −) must be given to the charge. If both $(V_B - V_A)$ and q are positive (or negative), the work done is positive. If $(V_B - V_A)$ and q have opposite signs, the work done is negative.

Absolute Potential: The absolute potential at a point is the work done against electric forces in carrying a unit positive test-charge from infinity to that point. Hence, the absolute potential at point-B is the difference in potential from A at ∞ to B.

Consider a point charge $q_\bullet$ in vacuum, and a point-P at a distance r from that point charge. The absolute potential at P due to the charge $q_\bullet$ is

$$V = k_0 \frac{q_\bullet}{r} \tag{25.2}$$

where $k_0 = 8.99 \times 10^9$ N · m^2/C^2 is the Coulomb constant for vacuum. The absolute potential at infinity (at $r = \infty$) is zero. As in Chapter 24, $k_0 = 1/4\pi\varepsilon_0$, and when a material medium surrounds the charge, k_0 must be replaced by $k = 1/4\pi\varepsilon$. See Table 24-1 for values of the permittivity in a sampling of materials.

Because of the superposition principle and the scalar nature of potential difference, the absolute potential at a point due to a number of point charges is

$$V = k_0 \sum \frac{q_{\bullet i}}{r_i} \tag{25.3}$$

where the r_i are the distances of the charges $q_{\bullet i}$ from the point in question. Negative $q_\bullet$'s contribute negative terms to the potential, while positive $q_\bullet$'s contribute positive terms.

The absolute potential due to a uniformly charged sphere, at points *outside* the sphere or *on* its surface, is $V = k_0 q/r$, where q is the charge on the sphere. This potential is the same as that due to a point charge $q_\bullet$ placed at the position of the sphere's center.

Electrical Potential Energy (PE_E): To carry a charge q from infinity to a point where the absolute potential is V, work in the amount qV must be done on the charge. This work appears as electrical potential energy (PE_E).

Similarly, when a charge q is carried through a **potential difference** V, work in the amount qV must be done on the charge. This work results in a change qV in the PE_E of the charge. For a potential *rise*, V will be positive and the PE_E will increase if q is positive. But for a potential *drop*, V will be negative and the PE_E of the charge will decrease if q is positive.

V **Related to** *E*: Suppose that in a certain region the electric field is uniform and is in the x-direction. Call its magnitude E_x. Because E_x is the force on a unit positive test-charge, the work done in moving the test-charge through a distance x is (from $W = F_x x$)

$$V = E_x x \tag{25.4}$$

The field between two large, parallel, oppositely charged, closely spaced metal plates is uniform. We can therefore use this equation to relate the electric field E between the plates to the plate separation d and their potential difference V. For parallel plates,

$$V = Ed \tag{25.5}$$

Electron Volt Energy Unit: The work done in carrying a charge $+e$ (coulombs) through a potential rise of exactly 1 volt is defined to be 1 **electron volt** (eV). Therefore,

$$1\ \mathrm{eV} = (1.602 \times 10^{-19}\ \mathrm{C})(1\ \mathrm{V}) = 1.602 \times 10^{-19}\ \mathrm{J} \tag{25.6}$$

Equivalently,

$$\text{Work or energy (in eV)} = \frac{\text{work (in joules)}}{e} \tag{25.7}$$

A Capacitor is a device that stores charge. The human body is a capacitor, albeit a poor one. Often, although certainly not always, a capacitor consists of two conductors separated by an insulator or dielectric (and that includes vacuum). The **capacitance** (C) of any capacitor is defined as

$$\text{Capacitance} = \frac{\text{Magnitude of charge on either conductor}}{\text{Magnitude of potential difference between conductors}} = \frac{q}{V} \tag{25.8}$$

For q in coulombs and V in volts, C is in **farads** (F). The farad is a very large capacitance; one usually works with microfarads ($1.00\ \mu\mathrm{F} = 1.00 \times 10^{-6}$ F) or nanofarads ($1.00\ \mathrm{nF} = 1.00 \times 10^{-9}$ F).

Parallel-Plate Capacitor: The capacitance of a parallel-plate capacitor whose opposing plate faces, each of area A, are separated by a small distance d is given by

$$C = K\varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d} \tag{25.9}$$

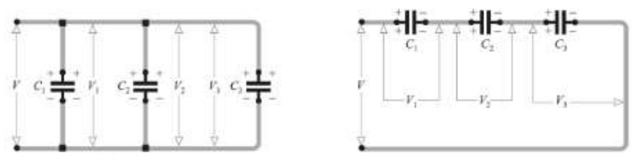
where $K\ \varepsilon/\varepsilon_0$ is the dimensionless dielectric constant (see Chapter 24) of the nonconducting material (the *dielectric*) between the plates, and

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} = 8.85 \times 10^{-12}\ \mathrm{F/m} \tag{25.10}$$

For vacuum, $K = 1$, so that a dielectric-filled parallel-plate capacitor has a capacitance K times larger than the same capacitor with vacuum between its plates. This result holds for a capacitor of arbitrary shape.

Equivalent Capacitance: Complicated circuits containing many capacitors can often be simplified by combining them in ways we shall discuss presently. For the especially simple circuits we will deal with, we can usually combine all the capacitors into one **equivalent capacitor** (C_{eq}). Such a capacitor has all the characteristics of the entire collection of capacitors; it stores the same charge and energy.

Capacitors in Parallel and Series: As shown in Fig. 25-1, capacitances add for capacitors in parallel, whereas reciprocal capacitances add for capacitors in series. When you are dealing with capacitors in series, it's convenient



$q = q_1 + q_2 + q_3$
$V = V_1 = V_2 = V_3$
$C_{eq} = C_1 + C_2 + C_3$

(*a*) Capacitors in parallel

$q = q_1 = q_2 = q_3$
$V = V_1 + V_2 + V_3$
$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

(*b*) Capacitors in series

Fig. 25-1

to use the $1/x$ key on your calculator. It is also helpful when you have two or more capacitors in series to keep in mind that for any two, C_1 and C_2,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \tag{25.11}$$

and so you can add them all two at a time.

Circuit elements (capacitors, resistors, batteries, etc.) have two **leads** or **terminals**. When three or more terminals are attached, the point (or region) where they attach is called a **node**. In Fig. 25-1(a) there are two nodes (the entire top and the entire bottom); in (b) there are none. In Fig. 25-1(a), *when both terminals of one circuit element are connected to both terminals of another element, they are in parallel*. When determining if two elements are in parallel, it does not matter how many terminals meet at each node. An unlimited number of elements can be arranged in parallel. *The same voltage*

V then appears across each of them.

In Fig. 25-1(*b*), *when one and only one terminal of one circuit element is connected to one and only one terminal of another element, they are in series*. If a node exists between capacitors, they are *not* in series. Be careful here, because as we simplify a circuit, nodes can disappear leaving elements in series. An unlimited number of elements can be arranged in series. *Capacitors in series carry the same charge no matter the value of their capacitance*.

A circuit will have at least two terminals leading to it, across which we will inevitably place a voltage source; that's the *V* in Fig. 25-1. Now examine Fig. 25-2 and notice that there are several terminals leading to the circuit: A, B, C, and D. The circuit can be approached using any pair of these terminals, but the equivalent capacitance will generally be different for each pair.

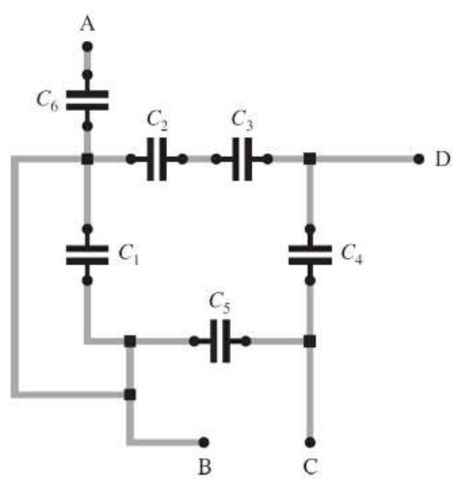


Fig. 25-2

If we want the equivalent capacitance across B-C, we see that C_6 is just hanging out. With no voltage across it, it does not affect the rest of the circuit and can be removed from the analysis. Similarly, terminal D can be removed. There is then no node between C_3 and C_4; they are in series. Notice that there is a wire across C_1; hence there is no voltage across it, and it too can be removed (you must leave the wire in place). We say that C_1 is **shorted** out.

With C_6 and C_1 removed in Fig. 25-2, the node where C_6, C_1, and C_2 met will vanish. Across B-C, that leaves C_2, C_3, and C_4 in series, and that resultant capacitance is in parallel with C_5. Notice that when we look across B-C, *the two wires representing B and C attach to the rest of the circuit at nodes*. Hence there are nodes on both sides of C_5, and so C_5 is not in series with anything else.

Energy Stored in a Capacitor: The energy (PE_E) stored in a capacitor of capacitance C that has a charge q and a potential difference V is

$$PE_E = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} \tag{25.12}$$

PROBLEM SOLVING GUIDE

Your calculator has a 1-over key designated as x^{-1} or $1/x$. It's very convenient when working with capacitors in series [Fig. 25-1(*b*)]. A common error is to compute $1/C$ for a string of series capacitors, and then to forget to take 1-over that to get C. When analyzing a circuit across a pair of terminals like B-C in Fig. 25-2, it's helpful to imagine a battery across those terminal. Draw the circuit labeling the nodes 1, 2, 3, etc. Find capacitors that are obviously in series and/or parallel and combine them. Every time you simplify it, redraw the circuit once again. Often you will have to work backward from the equivalent circuit to the original, so several drawings are a must. Remember that *the charge on the equivalent capacitor representing a string of series elements is the same on each of those series capacitors*.

SOLVED PROBLEMS

25.1 [I] In Fig. 25-3, the potential difference between the metal plates in air is 40 V. (*a*) Which plate is at the higher potential? (*b*) How much work must be done to carry a +3.0 C charge from *B* to *A*? From *A* to *B*? (*c*) How do we know that the electric field is in the direction indicated? (*d*) If the plate separation is 5.0 mm, what is the magnitude of $\vec{\mathbf{E}}$?

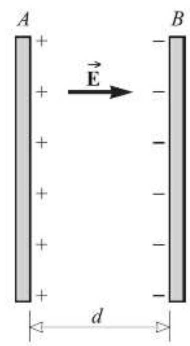


Fig. 25-3

(a) A positive test charge between the plates is repelled by A and attracted by B. Left to itself, the positive test charge will move from A to B, and so A is at the higher potential.

(b) The magnitude of the work done in carrying a charge q through a potential difference V is qV. Thus the magnitude of the work done in the present situation is

$$W = (3.0\ \text{C})(40\ \text{V}) = 0.12\ \text{kJ}$$

Because a positive charge between the plates is repelled by A, positive work (+120 J) must be done to drag the +3.0 C charge from B to A. To restrain the charge as it moves from A to B, negative work (−120 J) is done.

(c) A positive test-charge between the plates experiences a force directed from A to B and this is, by definition, the direction of the field.

(d) For closely spaced parallel plates, $V = Ed$. Therefore,

$$E = \frac{V}{d} = \frac{40\ \text{V}}{0.0050\ \text{m}} = 8.0\ \text{kV/m}$$

Notice that the SI units for electric field, V/m and N/C, are identical.

25.2 [I] How much work is required to carry an electron from the positive terminal of a 12-V battery to the negative terminal?

Going from the positive to the negative terminal, one passes through a potential drop. In this case it is $V = -12$ V. Then

$$W = qV = (-1.6 \times 10^{-19}\ \text{C})(-12\ \text{V}) = 1.9 \times 10^{-18}\ \text{J}$$

As a check, notice that an electron, if left to itself, will move from negative to positive because it is a negative charge. Hence, positive work must be done to carry it in the reverse direction as required here.

25.3 [I] How much electrical potential energy does a proton lose as it falls through a potential drop of 5 kV?

The proton carries a positive charge. It will therefore move from regions of high potential to regions of low potential if left free to do so. Its change in potential energy as it moves through a potential difference V is Vq. In our case, $V = -5$ kV. Therefore,

$$\text{Change in PE}_E = Vq = (-5 \times 10^3\ \text{V})(1.6 \times 10^{-19}\ \text{C}) = -8 \times 10^{-16}\ \text{J}$$

25.4 [II] An electron starts from rest and falls through a potential rise of 80 V. What is its final speed?

Positive charges fall through potential drops; negative charges, such as electrons, fall through potential rises.

$$\text{Change in PE}_E = Vq = (80\ \text{V})(-1.6 \times 10^{-19}\ \text{C}) = -1.28 \times 10^{-17}\ \text{J}$$

This lost PE_E appears as KE of the electron:

$$\text{PE}_E \text{ lost} = \text{KE gained}$$

$$1.28 \times 10^{-17}\ \text{J} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 - 0$$

and

$$v_f = \sqrt{\frac{(1.28 \times 10^{-17}\ \text{J})(2)}{9.1 \times 10^{-31}\ \text{kg}}} = 5.3 \times 10^6\ \text{m/s}$$

25.8 [III] The nucleus of a tin atom in vacuum has a charge of +50*e*. (*a*) Find the absolute potential *V* at a radial distance of 1.0×10^{-12} m from the nucleus. (*b*) If a proton is released from this point, how fast will it be moving when it is 1.0 m from the nucleus?

$$(a)\quad V = k_0 \frac{q}{r} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(50)(1.6 \times 10^{-19}\ \mathrm{C})}{10^{-12}\ \mathrm{m}} = 72\ \mathrm{kV}$$

(*b*) The proton is repelled by the nucleus and flies out to infinity. The absolute potential at a point is the potential difference between the point in question and infinity. Hence, there is a potential drop of 72 kV as the proton flies to infinity.

Usually we would simply assume that 1.0 m is far enough from the nucleus to consider it to be at infinity. But, as a check, compute *V* at *r* = 1.0 m:

$$V_{1\,\mathrm{m}} = k_0 \frac{q}{r} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(50)(1.6 \times 10^{-19}\ \mathrm{C})}{1.0\ \mathrm{m}} = 7.2 \times 10^{-8}\ \mathrm{V}$$

which is essentially zero in comparison with 72 kV.

As the proton falls through 72 kV,

$$\text{KE gained} = \mathrm{PE}_E \text{ lost}$$

$$\tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 = qV$$

$$\tfrac{1}{2}(1.67 \times 10^{-27}\ \mathrm{kg}) v_f^2 - 0 = (1.6 \times 10^{-19}\ \mathrm{C})(72\,000\ \mathrm{V})$$

from which $v_f = 3.7 \times 10^6$ m/s.

25.9 [II] The following point charges are placed on the *x*-axis in air: +2.0 μC at *x* = 20 cm, −3.0 μC at *x* = 30 cm, −4.0 μC at *x* = 40 cm. Find the absolute potential on the axis at *x* = 0.

Potential is a scalar, and so

$$V = k_0 \sum \frac{q_i}{r_i} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \left(\frac{2.0 \times 10^{6}\ \mathrm{C}}{0.20\ \mathrm{m}} + \frac{-3.0 \times 10^{-6}\ \mathrm{C}}{0.30\ \mathrm{m}} + \frac{-4.0 \times 10^{-6}\ \mathrm{C}}{0.40\ \mathrm{m}} \right)$$

$$= (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})(10 \times 10^{-6}\ \mathrm{C/m} - 10 \times 10^{-6}\ \mathrm{C/m} - 10 \times 10^{-6}\ \mathrm{C/m}) = -90\ \mathrm{kV}$$

25.12 [III] In Fig. 25-4, the medium is vacuum. Charge at *A* is +200 pC, while the charge at *B* is −100 pC. (*a*) Find the absolute potentials at points-*C* and -*D*. (*b*) How much work must be done to transfer a charge of +500 μC from point-*C* to point-*D*?

Fig. 25-4

$$(a)\ V_C = k_0 \sum \frac{q_i}{r_i} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \left(\frac{2.00 \times 10^{-10}\ \mathrm{C}}{0.80\ \mathrm{m}} - \frac{1.00 \times 10^{-10}\ \mathrm{C}}{0.20\ \mathrm{m}} \right) = -2.25\ \mathrm{V} = -2.3\ \mathrm{V}$$

$$V_D = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \left(\frac{2.00 \times 10^{-10}\ \mathrm{C}}{2.20\ \mathrm{m}} - \frac{1.00 \times 10^{-10}\ \mathrm{C}}{0.80\ \mathrm{m}} \right) = +7.88\ \mathrm{V} = +7.9\ \mathrm{V}$$

(*b*) There is a potential rise from *C* to *D* of $V = V_D - V_C = 7.88\ \mathrm{V} - (-2.25\ \mathrm{V}) = 10.13\ \mathrm{V}$. So

$$W = V_q = (10.13\ \mathrm{V})(5.00 \times 10^{-4}\ \mathrm{C}) = 5.1\ \mathrm{mJ}$$

SUPPLEMENTARY PROBLEMS

26.20 **[I]** How many electrons per second pass through a section of wire carrying a current of 0.70 A?

26.21 **[I]** An electron gun in a TV set shoots out a beam of electrons. The beam current is 1.0×10^{-5} A. How many electrons strike the TV screen each second? How much charge strikes the screen in a minute?

26.22 **[I]** What happens to the resistance of a copper wire if its length is doubled, all else kept constant?

26.23 **[I]** What happens to the resistance of a copper wire if its diameter is doubled, all else kept constant?

26.33 **[I]** A dry cell delivering 2 A has a terminal voltage of 1.41 V. What is the internal resistance of the cell if its open-circuit voltage is 1.59 V?

26.34 **[II]** A cell has an emf of 1.54 V. When it is in series with a 1.0-Ω resistance, the reading of a voltmeter connected across the cell terminals is 1.40 V. Determine the cell's internal resistance.

26.35 **[I]** The internal resistance of a 6.4-V storage battery is 4.8 mΩ. What is the theoretical maximum current on short circuit? (In practice the leads and connections have some resistance, and this theoretical value would not be attained.)

26.36 **[I]** A battery has an emf of 13.2 V and an internal resistance of 24.0 mΩ. If the load current is 20.0 A, find the terminal voltage.