ELECTROSTATICS

Electrostatics is the study of charges at rest.

The structure of the atom

An atom has a massive nucleus housing the protons and the neutrons. Electrons, for now, we shall assume move round the nucleus. Neutrons are neutral. Protons are positively charged and electrons negatively charged. The charge on the proton is equal to that on the electron, except that the electron carries a negative charge while the proton carries a positive charge. In a neutral atom, the number of electrons is equal to the number of protons. Hence, a neutral atom has no charge. To charge a body negative, we add electron. To charge an atom positive, we remove electrons. Why do we deal with electrons in both cases? It is easier to remove and add electrons since they are not in the nucleus. Pulling a proton out of the nucleus requires an extremely large force due to the nuclear forces holding the nucleus together. Like charges repel and unlike charges attract. As such, if we bring two bodies with like charges close, they suffer mutual repulsion. When they are oppositely charged, they attract.

Conductors, Semiconductors and Insulators

Conductors are materials that have free electrons in their outermost shells. Examples include all metals. When an electric field is applied, these free electrons move, allowing current to flow in a metal. Insulators do not have any free electrons in their outermost shells. As such, when you apply an electric field across an insulator, the electrons are only displaced, and you say such a material is polarized. An insulator can break down if the electric field is large enough. A good example is air. When the electric field is large enough, electric current is able to pass through, producing lightning and thunder. Some materials are in-between conductors and insulators. These are semiconductors. The outermost shell has four electrons and such atoms bond via covalent bonds. Such materials conduct little electricity when the temperature increases from zero Kelvin. When doped by the appropriate dopant, a semiconductor becomes a good conductor of electricity. Conductors have a high electric conductivity. Insulators have low electric conductivity and the conductivities of semiconductors are in-between. You can also see why metals are also good conductors of heat. Free electrons get heated and move towards the cooler part of the metallic material. Conductors of heat have high thermal conductivity. Insulators have low thermal conductivity.

Polarisation

When you bring a positively charged body closer to a neutral body, electrons in the neutral body are attracted towards the incoming positively charged body. This leaves the other side of the neutral body is then left with an excess positive charge. The neutral body is then said to be polarised. This kind of polarisation is different from polarisation under optics. The neutral body is attracted towards the incoming positively charged body. If the charged body is negatively charged, electrons in the neutral body are repelled to the other end of the neutral body. Then there is a net positive charge closer to the incoming negative charge. Yet again, there is attraction. Thus, attraction is not proof two bodies are charged. It could be that the bodies are both charged, but with opposite signs, or only one of the bodies is charged. This is why when you run a comb through your hair during the dry season, it is able to pick up small pieces of paper. This is because the comb becomes charged while running through your hair. Bringing the comb close to a neutral piece of paper causes the paper to become polarised and the comb is able to attract the piece of paper.

Methods of Charging
Charging by Contact/Conduction

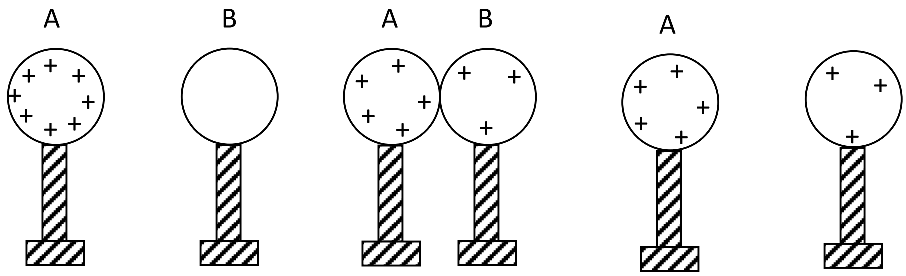


Fig. 1.1: Charging by contact

In Fig. 1, a charged sphere mounted on an insulator, A, is brought in contact with an uncharged sphere also mounted on an insulator, B. The charge on A is redistributed on A and B. A is then pulled away. The uncharged sphere is now left with the charge it got from A.

1.4.2 Charging by Induction

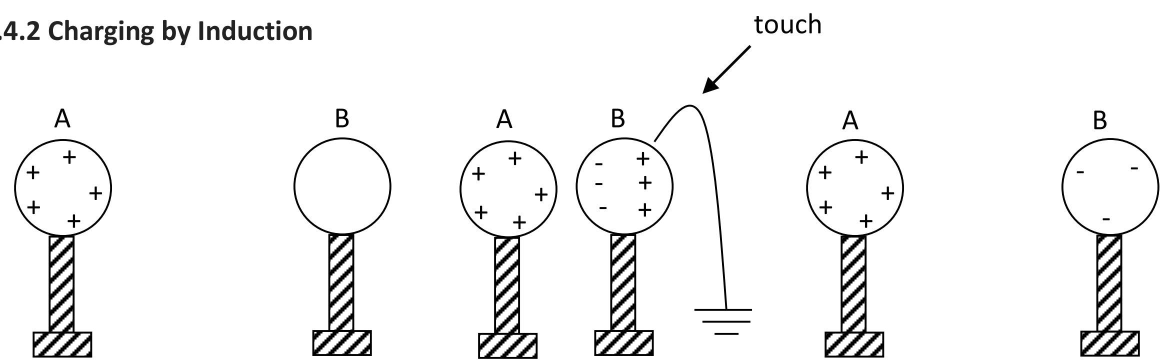


Fig. 1.2: Charging by induction

In this case, an uncharged sphere, A, is brought close to an uncharged sphere B, both mounted on insulators. The uncharged sphere becomes polarised, with the unlike charge, in this case, negative charge closer to the positively charged sphere. When B is earthed, electrons from the earth flow in to neutralise the positive charge. Upon separation, B has become negatively charged. If A had been a negatively charged sphere, B would have acquired a positive charge.

Note that in charging by contact, B has the same charge as A. In the case of charging by induction, B has an unlike charge.

Charging by Friction/Rubbing

Charging by contact is also called charging by conduction. When two bodies rub against each other, electrons might be lost by one and gained by the other. The body that has a greater affinity for electron out of the two bodies takes away electrons from the other body, becoming negatively charged in the process while the other body becomes positively charged. This is charging by friction or rubbing, a process called triboelectricity.

The triboelectric series is a series that shows which materials have greater affinity for electrons. Materials that are lose electrons to the other material are higher on the table. We list some materials in the triboelectric series:

The Triboelectric Series
Asbestos
Rabbit's fur
Glass
Mica
Wool
Cat's fur
Silk
Human skin
Cotton
Steel
Wood
Amber
Ebonite
Rubber

It follows that if you rub glass with wool, glass will lose electrons to wool. Glass becomes positively charged and wool becomes negatively charged.

The Gold-leaf Electroscope

Production of Charges
The Van-de-Graaf Generator

The Electrophorus

CHAPTER TWO
ELECTRIC FORCE AND FIELD

2.1 Electric force between 2 charges, Coulomb's Law

Coulomb's law states that the electrostatic force between a pair of charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F \propto \frac{Q_1 Q_2}{r^2} \qquad 2.1$$

where Q_1 and Q_2 are the charges and r is the distance between them.

Hence, we can write

$$F = \frac{kQ_1 Q_2}{r^2} \qquad 2.2$$

k is a constant of proportionality, which in a vacuum is equal to $\frac{1}{4\pi\varepsilon_0}$, where ε_0 is the permittivity of free space, equal to $8.854\times10^{-12} F/m$. The Farad is the unit of capacitance. $\frac{1}{4\pi\varepsilon_0}$ is approximately equal to $9\times10^{9} m/F$.

We can write

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} \qquad 2.3$$

For like charges, the product Q_1Q_2 is positive. A positive force is repulsive. The force on Q_2 is equal and opposite to that on Q_1, according to Newton's Third Law of motion. The charges experience mutual repulsion. For unlike charges, the product Q_1Q_2 is negative. A negative force is attractive. The charges experience mutual attraction. Like charges repel, unlike charges attract.

Notice that when the distance between the charges is halved, the force between them is quadrupled,

$$\frac{kQ_1Q_2}{r^2} \to \frac{kQ_1Q_2}{(r/2)^2} = 4\frac{kQ_1Q_2}{r^2}$$

Reduced to a third,

$$\frac{kQ_1Q_2}{r^2} \to \frac{kQ_1Q_2}{(r/3)^2} = 9\frac{kQ_1Q_2}{r^2}$$

If doubled,

$$\frac{kQ_1Q_2}{r^2} \to \frac{kQ_1Q_2}{(2r)^2} = \frac{1}{4}\frac{kQ_1Q_2}{r^2}$$

When tripled,

$$\frac{kQ_1Q_2}{r^2} \to \frac{kQ_1Q_2}{(3r)^2} = \frac{1}{9}\frac{kQ_1Q_2}{r^2}$$

Example

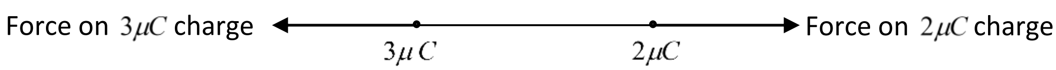
Calculate the electrostatic force between the proton and the electron in a hydrogen atom. You may take the atomic radius as 10^{-14} m. The charge on the electron is $-1.6\times10^{-19} C$ and that on the proton $1.6\times10^{-19} C$.

$$F = \frac{kQ_1Q_2}{r^2} = \frac{9\times10^9 \times1.6\times10^{-19} \times(-1.6\times10^{-19})}{(10^{-14})^2} = -2.304\times10^{-9}\,N$$

Force is a vector. As such, if two forces act on a body, the forces must be added vectorially.l

Example

Two charges magnitude $3\mu C$ and $2\mu C$ are on the x-axis, with the $3\mu C$ on the left. If the distance between them is 0.2 mm, calculate the force (magnitude and direction) on each charge.



$$F = \frac{kQ_1Q_2}{r^2} = \frac{9\times10^9 \times3\times10^{-6} \times2\times10^{-6}}{(0.2\times10^{-3})^2} = \frac{54\times10^{-3}}{0.04\times10^{-6}} = 1350\times10^3 = 1.35\times10^6\,N$$

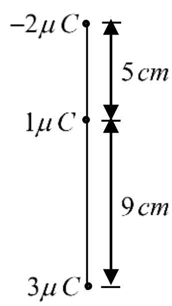
There is a $1.35\times10^6\,N$ on each charge. The force on the $3\mu C$ charge is to the left while that on the $2\mu C$ is to the right. We can also write this in terms of unit vector $\mathbf{i}$ as $(-1.35\times10^6\,N)\mathbf{i}$ and $(1.35\times10^6\,N)\mathbf{i}$ respectively.

Example

Three charges $-2\mu C$, $1\mu C$ and $3\mu C$ are arranged on the y-axis as shown in the figure below. Calculate the force on

(i) the $1\mu C$ charge

(ii) the $3\mu C$ charge



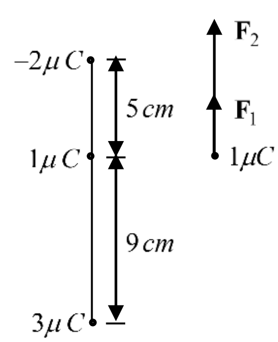
(i) Let $\mathbf{F}_1$ be the force on the $1\mu C$ due to the $-2\mu C$ charge and $\mathbf{F}_2$ the force on the $1\mu C$ due to the $3\mu C$. The magnitude of the forces are,

$$F_1 = \frac{9\times10^9 \times1\times10^{-6} \times2\times10^{-6}}{(5\times10^{-2})^2} = 7.2\,N$$

$$F_2 = \frac{9\times10^9 \times1\times10^{-6} \times3\times10^{-6}}{(9\times10^{-2})^2} = 3.33\,N$$

The two forces are in the same direction, and hence, add up.

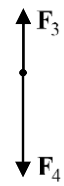
The net force the $1\mu C$ charge is $\mathbf{F}_1 + \mathbf{F}_2 = (7.2\,N)\mathbf{j} + (3.33\,N)\mathbf{j} = (10.53\,N)\mathbf{j}$



(ii) Let $\mathbf{F}_3$ be the force on the $3\mu C$ charge due to the $-2\mu C$ charge and $\mathbf{F}_4$ the force on the $3\mu C$ due to the $1\mu C$ charge.

$$F_3 = \frac{9\times10^9 \times2\times10^{-6} \times3\times10^{-6}}{(14\times10^{-2})^2} = 1.93\,N$$

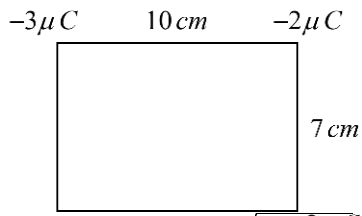
$$F_4 = \frac{9\times10^9 \times 1\times10^{-6} \times 3\times10^{-6}}{(9\times10^{-2})^2} = 3.33\,N$$



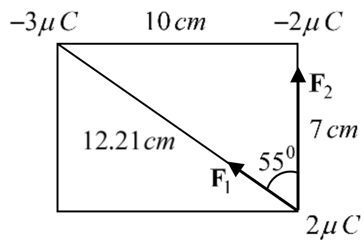
The net force the $3\mu C$ charge is $\mathbf{F}_3 + \mathbf{F}_4 = (1.93\,N)\mathbf{j} + (-3.33)\mathbf{j} = (-1.4N)\mathbf{j}$

Example

Calculate the force on the $2\mu C$ charge in the figure below.



The distance between the $2\mu C$ and the $-3\mu C$ is $\sqrt{10^2+7^2} = \sqrt{149} = 12.21\,cm$. $\theta = \tan^{-1}\left(\frac{10}{7}\right) = 55^0$



$$F_1 = \frac{9\times10^9 \times 3\times10^{-6} \times 2\times10^{-6}}{(12.21\times10^{-2})^2} = \frac{9\times10^9 \times 3\times10^{-6} \times 2\times10^{-6}}{149\times10^{-4}} = 3.6N$$

$$F_2 = \frac{9\times10^9 \times 2\times10^{-6} \times 2\times10^{-6}}{(7\times10^{-2})^2} = 7.35\,N$$

The components of the resultant force are:

x-component: $3.6\sin 55^0 = 2.95N$

y-component: $7.35 + 3.6\cos 55^0 = 9.41\,N$

The resultant is $\sqrt{2.95^2 + 9.41^2} = \sqrt{97.25} = 9.86\,N$

Example

The distance between two charges is doubled while the magnitude of one of them is tripled. What is the ratio of the new force between them to the old force?

$$F = \frac{kQ_1Q_2}{r^2} \rightarrow \frac{k3Q_1Q_2}{(2r)^2} = \frac{3}{4}\frac{kQ_1Q_2}{r^2} \Rightarrow F_2 = \frac{3}{4}F_1 \Rightarrow \frac{F_2}{F_1} = \frac{3}{4}$$

2.2 Electric Field

Qualitatively, the electrostatic or electric field of a charge is the region around it, within which its effect is felt. In other words, it is the region around it within which another charge will experience a force due to the charge, or conversely, the region in which the charge under consideration will be affected by any charge placed in the region.

The quantitative definition of electrostatic field at a point due to a particular charge is the electrostatic force experienced due to the charge per unit charge placed at the point.

$$\mathbf{E} = \frac{\mathbf{F}}{q} \qquad 2.4$$

where q is the charge placed in the field of the charge under consideration, which we shall call Q. The unit of the electric field is N/C. In magnitude,

$$E = \frac{\dfrac{Qq}{4\pi\,\varepsilon_0 r^2}}{q} = \frac{Q}{4\pi\,\varepsilon_0 r^2} \qquad 2.5$$

The charge q is called the test charge, and as you can see from equation ( ), it does not come into the equation for the electric field of charge Q. Hence, it can be made as small as possible. This is because it is desirable not to distort the field we are trying to measure. Indeed, the electrostatic force experienced by a pair of charges is due to the interaction between the fields of the charges. We therefore, more precisely, write

$$\mathbf{E} = \lim_{q \to 0} \frac{\mathbf{F}}{q} \qquad 2.6$$

We notice that if Q is positive, then the electric field is positive. Hence, the lines representing it, called the field lines point away from it. If Q is negative, the electric field is negative, and the lines representing it point inwards towards it. (Fig. 2.1).

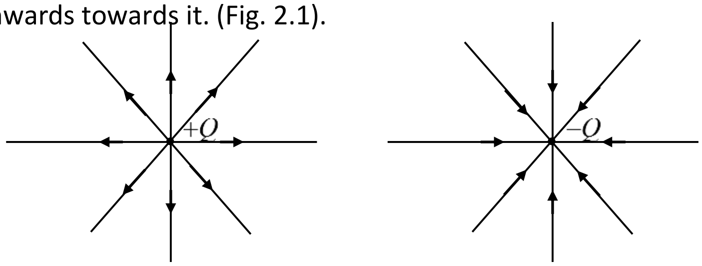


Fig. 2.1: Electric fields of positive and negative charges

The electric field of like charges

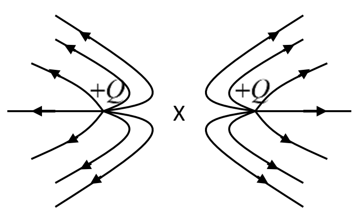


Fig. 2.2: Like charges repel, both charges positive

X is the neutral point, a point where the two electric fields (oppositely directed) are equal in magnitude. The electric field at that point is zero.

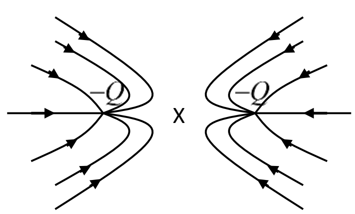


Fig. 2.3: Like charges repel, both charges negative

X is yet again the neutral point.

The electric field of unlike charges

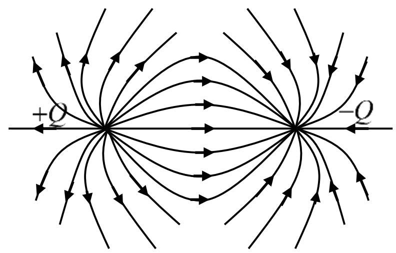


Fig. 2.4: Unlike charges attract

From equation (2.4),

$$\mathbf{F} = q\mathbf{E} \qquad 2.7$$

Once we calculate the electric field at a point, we can then calculate the electric force it experiences due to the electric field at that point.

If the charge q is positive, the electric force is in the direction of the electric field. If q is negative, the electric force is in a direction opposite that of the electric field.

Example

Calculate the magnitude and direction of a $5\mu C$ charge at a point 3 cm from the charge. Hence, calculate the electrostatic force on a $-2\ \mu C$ at that point.

A charge of this nature is said to be a point charge. Notice that so far, we have only dealt with point charges. The field of a point charge points away from the charge or towards it in a radial direction, depending on whether it is positive or negative.

$$E = \frac{Q}{4\pi\,\varepsilon_0 r^2} = \frac{9\times10^9 \times 5\times10^{-6}}{(3\times10^{-2})^2} = 5\times10^7\ N/C$$

Since it is a positive charge, it is directed away from the charge.

The electric force the $-2\ \mu C$ experiences is,

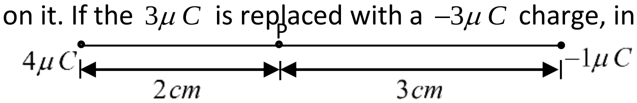
$$\mathbf{F} = q\mathbf{E} = -2\times10^{-6}\,C\times5\times10^7\ N/C = -10^8\ N \text{ (direction explained below)}$$

This force is towards the force that created the electric field, since the sign is negative. The charge that created the field is positive and unlike charges attract. The two charges experience mutual attraction.

Force is a vector. As such, if two forces act on a body, the forces must be added vectorially.

Example

Calculate the electric field between two charges $4\mu C$ and $-1\mu C$ charge at a point P, 2 cm from the $4\mu C$ charge, when the charges are separated by 5 cm. If a $3\mu C$ is placed at P, calculate the magnitude of the electric force on it. If the $3\mu C$ is replaced with a $-3\mu C$ charge, in what direction will the $-3\mu C$ charge move?



Let the electric field due to the $4\mu C$ and the $-1\mu C$ at P be, respectively, $\mathbf{E}_1$ and $\mathbf{E}_2$.

$$E_1 = \frac{9\times10^9 \times 4\times10^{-6}}{(2\times10^{-2})^2} = 9\times10^7\ N/C$$

$$E_2 = \frac{9\times10^9 \times 1\times10^{-6}}{(3\times10^{-2})^2} = 10^7\ N$$

The two fields point in the same direction, as the field due to the $4\mu C$ charge pushes outwards and that due to the $-1\mu C$ charge pulls inward. The field at P is therefore towards the $-1\mu C$ charge. Its magnitude is the sum of the two fields as indicated in Fig. , equal to

$$9\times10^9\ N/C + 10^9\ N/C = 10^{10}\ N/C$$

We could also add these forces as follows

$$(9\times10^9\ N/C)\mathbf{i} + (10^9\ N/C)\mathbf{i} = (10^{10}\ N/C)\mathbf{i}$$

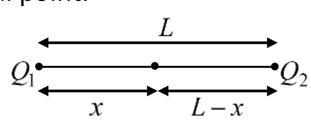
The $3\mu C$ will experience a force,

$\mathbf{F} = q\mathbf{E} = 3\times10^{-6} C\times(10^{10}\ N/C)\mathbf{i} = (3\times10^4\ N)\mathbf{i}$, directed to the right, towards the $-1\mu C$ charge.

If replaced with the $-3\mu C$ charge, the force on it will be equal as in the case of the $3\mu C$, but in the opposite direction.

2.2.1 Neutral Points

The fields between two like charges are in opposite directions. This means that somewhere between the two charges, the net field is zero. This is the neutral point. Refer to Figs again, and you will see that only in such a situation do you have a neutral point.



$$\frac{Q_1}{4\pi\varepsilon_0 x^2} = \frac{Q_2}{4\pi\varepsilon_0 (L-x)^2} \Rightarrow Q_1(L-x)^2 = Q_2 x^2 \Rightarrow Q_1 L^2 - 2LxQ_1 + Q_1 x^2 = Q_2 x^2$$

Rearranging,

$$(Q_1 - Q_2)x^2 - 2LQ_1 x + Q_1 L^2 = 0$$

Using the quadratic formula,

$$x = \frac{-(-2LQ_1) \pm \sqrt{(-2LQ_1)^2 - 4(Q_1-Q_2)Q_1L^2}}{2(Q_1-Q_2)} = \frac{2LQ_1 \pm \sqrt{4L^2Q_1{}^2 - 4Q_1{}^2L^2 + 4Q_1Q_2L^2}}{2(Q_1-Q_2)}$$

$$= \frac{2LQ_1 \pm \sqrt{4Q_1Q_2L^2}}{2(Q_1-Q_2)} = \frac{2LQ_1 \pm 2L\sqrt{Q_1Q_2}}{2(Q_1-Q_2)} = \frac{LQ_1 \pm L\sqrt{Q_1Q_2}}{Q_1-Q_2} = \frac{L(Q_1 \pm \sqrt{Q_1Q_2})}{Q_1-Q_2}$$

Example

Calculate the distance of the neutral point from a $2\mu C$ charge and a $5\mu C$ charge separated by a distance of 4 mm.

$$\frac{Q_1}{4\pi\varepsilon_0 x^2} = \frac{Q_2}{4\pi\varepsilon_0 (4-x)^2} \Rightarrow Q_1(4-x)^2 = Q_2 x^2 \Rightarrow 2(4-x)^2 = 5x^2 = 2(x^2 - 8x + 16) = 2x^2 - 16x + 32$$

$$3x^2 + 16x - 32 = 0 \Rightarrow x = \frac{-16 \pm \sqrt{(-16)^2 - 4(3)(-32)}}{2(3)} = \frac{-16 \pm \sqrt{256+384}}{6}$$

$$x = \frac{-16 \pm \sqrt{640}}{6} = \frac{-16 \pm 25.3}{6} = \frac{9.3}{6} = 1.55\,cm$$

Alternatively,

$$\frac{LQ_1 \pm L\sqrt{Q_1 Q_2}}{Q_1 - Q_2} = \frac{L(Q_1 \pm \sqrt{Q_1 Q_2})}{Q_1 - Q_2} = \frac{4(2 \pm \sqrt{2 \times 5})}{2-5} = -\frac{4}{3}(2 \pm 3..16) = -\frac{4}{3}(-1.16) = 1.54\,cm$$

Note that we must take the positive result as distance cannot be negative.

Electric Field Line and Electric Field Vector

The lines we drew in Figs. are called electric field lines. An electric field line is an imaginary line drawn in space such that it is tangent at every point to the electric field vector. Electric field lines go from positive to negative charges. Hence, a positive charge moves in the direction of the electric field while a negative charge moves opposite the direction of the electric field. It might be helpful to imagine the positive charge moving to a negative charge attracting it to the point where the field line ends, and a negative charge attracted to the positive charge from which the field line arises. This is also why **field lines do not cross**. If they do, that would mean that a charge would be 'confused' as there are two or more directions in which it can move.

Bear in mind: the electric field vector is tangential to the electric field line.

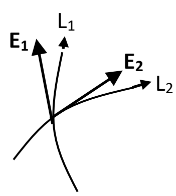
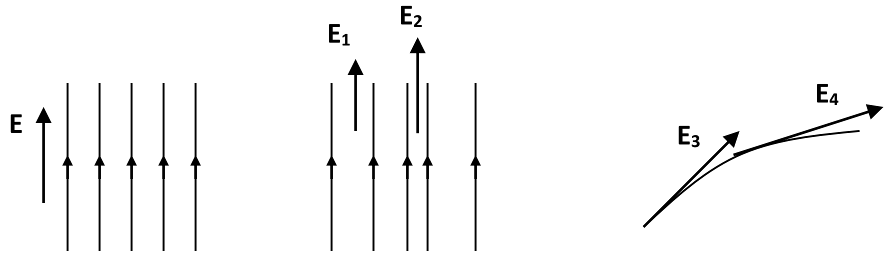


Fig. 2.5: Field lines do not cross

In Fig 2.5, there are two field lines $L_1$and L_2. The electric field lines cross at the point indicated in the diagram. $\mathbf{E_1}$ is tangential to L_1 and $\mathbf{E_2}$ is tangential to L_2. If this were to be the case, it would mean that the electric field at the point points in two different directions even if the magnitudes are the same. It would also mean that a positive charge placed at the point would not know in which direction it should move. Neither would a negative charge. This is why field lines do not cross.

Uniform and Non-Uniform Electric Fields

A vector field is a quantity in space whose quantity at every point is a vector. A uniform vector field, such as a uniform electric field vector, must be uniform both in magnitude and direction. A vector is uniform only if it is uniform in both magnitude and direction. Constant in magnitude would mean that the field lines are evenly spaced. Constant in direction implies the field vectors are straight. You can now see that the electric field of a point charge is not uniform. It radiates outwards from the charge.



Uniform field:
lines straight,
evenly spaced

Non-uniform field:
lines straight,
unevenly spaced

Non-uniform field:
lines not straight,
Even if same magnitude, directions different

(a) (b) (c)

Fig. 2.6: Uniform and non-uniform fields

In Fig. 2.6a, the field lines are straight, parallel and evenly spaced. The evenness of the lines means the magnitude of the electric field is constant throughout the region. The field lines point in the same direction, being tangent to each line. This is a uniform electric field. That is why we have only drawn one representative electric field **E**.

In Fig. 2.6b, the field lines though straight, are not evenly spaced. The directions of the electric fields are the same, but the magnitudes are not, being higher where the lines are more closely spaced. This is a non-uniform field on account of the unevenness of the lines.

In Fig. 2.6c, though there is only one field line, it is not straight. The electric field does not have a fixed direction throughout the line. It is tangential to the line at each point. For that reason, the direction of the field is not constant. Even if the magnitude is constant, the field cannot be uniform. Remember this is also the reason a body moving on a curve cannot be in uniform motion even if the speed is constant. A vector is constant only if it is constant in magnitude and in direction.

Electric Flux

The electric flux through a surface is the number of field lines crossing the surface normally. We define the vector area as having the magnitude of the area and a direction normal out of the surface (Fig. a). We then define the electric or electrostatic flux out of surface as

$$\psi = EA\cos\theta \qquad 2.8$$

where E is the magnitude of the electric field, A is the magnitude of the area and θ the angle between the normal to the area and normal out of the area.

Vectorially, we would write this as,

$$\psi = \mathbf{E}\cdot\mathbf{A} \qquad 2.9$$

We can define the area normal to the electric field as $A_n = A\cos\theta$. If $\theta = 0$, the entire area is normal to the electric field, $A_n = A$. If the area is parallel to the electric field, then $\theta = 90^0$ as the normal to the surface will then be perpendicular to the electric field, the normal area, $A_n = A\cos 90^0 = 0$.

Hence, we can write equation (2.8) as,

$$\psi = EA_n \qquad 2.10$$

Fig. 2.7a is the general representation of the electric flux out of a surface for any given situation. In Fig. 2.7b, the electric field is parallel to the surface of the area. This is perpendicular to the normal to the surface. Hence, $\theta = 90^0$, and

$$\psi = EA\cos 90^0 = 0 \qquad 2.11$$

An electric field parallel to the surface contributes no electric flux through the surface.

In Fig. 2.7c, the electric field is perpendicular to the surface, coming out of the surface (tip of an arrow). $\theta = 0^0$, and

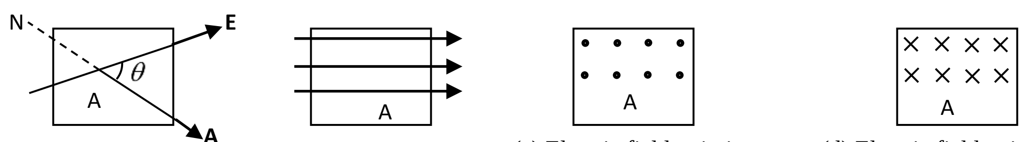
$$\psi = EA\cos 0^0 = EA \qquad 2.12$$

Flux out of a surface is positive.

In Fig. 2.7d, the electric field is perpendicular to the surface, going into the surface (base of an arrow). $\theta = 180^0$, and

$$\psi = EA\cos 180^0 = -EA \qquad 2.13$$

Flux into a surface is negative.



(a) Electric flux into surface = $EA\cos\theta$

(b) Electric Flux through A = 0 $\mathbf{E} \perp \mathbf{A}, \theta = 90^0$

(c) Electric field pointing out of surface, along the normal Electric flux through area is EA $\mathbf{E} \parallel \mathbf{A}, \theta = 0^0$

(d) Electric field pointing out of surface, along the normal Electric flux through area is $-EA$ $\mathbf{E} \parallel \mathbf{A}, \theta = 180^0$

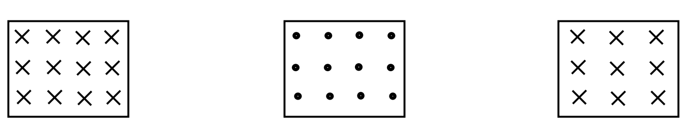
Fig. 2.7: Electric flux in general and in some given cases

Electric field is also called electric field intensity or electric field strength.

From equations (2.8) and (2.10), we can write

$$E = \frac{\psi}{A\cos\theta} = \frac{\psi}{A_n} \qquad 2.14$$

Hence, electric field is the electric flux per unit normal area. This is another way of defining electric field, one that is linked to the area the field passes through. We assume the areas in Fig. are all equal. The electric flux per unit area is the same for Figs. 2.8a and 2.8b, except that the directions of the electric field are opposite. In Fig 2.8c, the electric field is less than that of Fig. 2.8a, as the electric flux per unit area is smaller. In conclusion, electric field is proportional to the number of field lines per unit normal area.



(a) Flux into surface (b) Flux out of surface (c)Less flux into surface

Fig. 2.8: More on the electric field in and out of a surface

From $\psi = EA_n$, we see that the electric flux is proportional to the electric field and the normal area. If for instance, the electric field is uniform, the electric flux is proportional only to the normal area. If the normal area is constant, then the flux is proportional to the electric field – more electric field lines implies more flux.

Gauss's Law

Gauss' law states that the electric flux out of any closed surface is equal to the net charge it encloses, is proportional to the net charge it encloses.

$$\psi \propto q \tag{2.15}$$

For free space, $\frac{1}{\varepsilon_0}$ is the constant of proportionality.

$$\psi = \frac{q}{\varepsilon_0} \tag{2.16}$$

If the surface encloses more than one charge,

$$\psi = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i$$

If the charge (if only one) or the net charge is negative, the electric flux points into the surface.

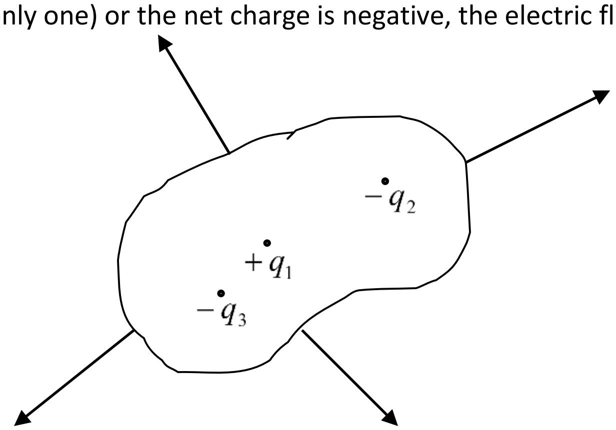


Fig. 2.9: Flux out of a surface enclosing discrete charges

$$\psi = \frac{1}{\varepsilon_0}(q_1 - q_2 - q_3)$$

A spherical surface encloses 4 charges, 2, 4, – 3, and – 5 μC . Calculate the electric flux out of the surface.

$$\psi = \frac{1}{\varepsilon_0}(2 + 4 - 3 - 5)\mu C = -\frac{2}{\varepsilon_0}Vm$$

The electric flux in this case is into the surface. The volt-metre is the unit of electric flux.

If the surface does not enclose any particular charge, such a charge does not contribute to the electric flux out of the surface. This is because the field of the charge crosses the surface equal times in and out. (Fig. 2.10).

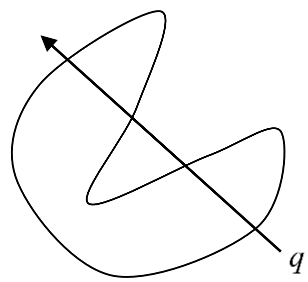


Fig. 2.10: Charge outside a surface does not contribute to the electric flux out of the surface

3. WORK DONE IN AN ELECTROSTATIC FIELD

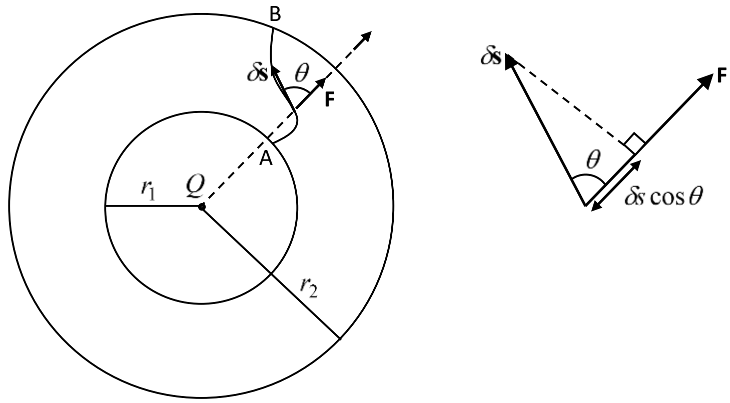


Fig. 3.1: Work done in an Electric field

In Fig.3.1, a charge q is moved from point A to point B in the electric field of the positive charge Q. The force on the charge is not constant as it is dependent on the distance from Q. We therefore have to chop the journey up into small bits of displacements over which we can assume that the force is approximately constant. The work done over a small element of displacement δs is as shown in the diagram,

$$\delta W = F\delta s\cos\theta \qquad 3.1$$

But $\delta s\cos\theta$ is along the radius. So, we can say $\delta s\cos\theta$. Therefore,

$$\delta W = F\delta r \qquad 3.2$$

We add up all the elements of work to make the total work done in moving the charge from A to B.

$$W = \sum \delta W = \sum F\delta r \qquad 3.3$$

In the limit as $\delta r \to 0$,

$$W = \int_A^B F dr = \int_{r_A}^{r_B} \frac{Qq}{4\pi\varepsilon_0 r^2} dr = \left[-\frac{Qq}{4\pi\varepsilon_0 r} \right]_{r_A}^{r_B} = -\frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = -\frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad 3.4$$

This is the work done by the field. The work done by the environment, which is the work we usually refer to, is the negative of this. Hence, the work done by the environment is,

$$W = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad 3.5$$

You can see that the work done depends only on the initial and the final points, and not on the path taken. Recall this is a characteristic of a conservative field. Electrostatic field is a conservative field, which means that the mechanical energy is conserved. It also means there is a potential. A potential is defined when the work done does not depend on the path taken. In another interpretation, the work done in a cycle is zero. That is, if the initial point is the same as the final, the work done is zero. This also means that the work done in taking the charge from A to B is equal but of a negative sign of the work done in moving it from B back to A.

Another conclusion we can draw is that if we move a charge q long an arc of a circle centred on the charge Q, or indeed along a sphere centred on Q, then the work done is zero, since the radius of the sphere is constant. This means that the force must be perpendicular to the displacement. The displacement is along the surface of the sphere or the arc of a circle. Hence, the force must be perpendicular, which means it must intersect such a surface at right angle. Such a surface is called an equipotential surface. We shall see more about this later.

Electric Potential Energy

The work done in moving a charge from a point to another is stored in it as potential energy. Let q as in Section be negative. Taking it from A to B in the field of the positive charge will be positive work done by us on the charge, as we have to force it to move away from the unlike charge. As soon as we release it upon pushing it to B, it moves along the radius to Q, if the latter is much more massive. They two charges actually move towards each other due to the mutually attractive force. That means it has been given a potential energy with which it can do work by moving to Q. If we keep Q stationary, by the time the charge q gets to the radius where it was before, the work done in moving it from A to B has been reversed. The charge has done positive work while we have done negative work. On its way to B, the charge had done negative work while we were doing positive work.

$$EPE = \frac{Qq}{4\pi\,\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \tag{3.6}$$

Example

(i) Calculate the work done in moving a $2\mu C$ charge from a radial distance of 2 mm to a radial distance 5 mm in the electrostatic field of a $3\mu C$ charge.

(ii) Calculate the work done in moving the charge back to its original position.

(iii) What is the work done in a cycle?

$$EPE = \frac{Qq}{4\pi\,\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = 9\times10^9\times3\times10^{-6}\times2\times10^{-6}\left[\frac{1}{5\times10^{-3}} - \frac{1}{2\times10^{-3}}\right] = 54\times10^{-3}\times10^3\left[0.2-0.5\right] = -1.62\,J$$

Notice that the work done by us is negative. This is because the charge 'willingly' goes. It is like when descending a staircase or walking downhill. The work you do is negative while the work done by gravity is positive. In this case, the work done by the field is positive. The field actually pushes the charge away.

$$EPE = 9\times10^9\times3\times10^{-6}\times2\times10^{-6}\left[\frac{1}{2\times10^{-3}} - \frac{1}{5\times10^{-3}}\right] = 54\times10^{-3}\times10^3\left[0.5-0.3\right] = 1.62\,J$$

This time around, we have to push it against the mutual repulsive force. We are doing positive work while the field is doing negative work.

(iii) The work done in a complete cycle is – 1.62 + 1.62 = 0 J.

Note that positive work results in positive electrostatic potential energy.

Electric Potential

The electric potential due to a given charge at a point is work done in moving a point charge from infinity up to that point.

$$V(r) = \frac{\frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right)}{q} = \frac{Q}{4\pi\varepsilon_0 r} \qquad 3.7$$

where we have taken $r_B = r$.

The unit of electrostatic potential is the Joule per Coulomb, J/C or the Volt (V).

Electrostatic potential energy and electrostatic potential are both scalars.

Example

Calculate the electrostatic potential due to a $10\mu C$ charge at a point 20 cm from it.

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{9\times10^9 \times 10\times10^{-6}}{20\times10^{-2}} = 4.5\times10^5 V$$

For a positive charge Q, the electrostatic potential is positive and becomes smaller as you move away from the charge. This means that the electrostatic potential increases as you approach the charge. In Fig. then, $V_B < V_A$ and $V_B - V_A < 0$. This means if you move a positive charge from A to B, then, $q(V_B - V_A) < 0$. If q is negative, $q(V_B - V_A) > 0$. That makes sense, right? Moving a positive charge away from another positive charge is easy. You are doing negative work, but moving a negative charge away from a positive charge is real work for you as you are working against the mutual attraction of the charges.

For a negative charge Q, the electrostatic potential is negative and becomes less negative as you move away. This means that the electrostatic potential decreases as you approach the charge.

We can now write equation ( ) as

$$W = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = q\left[\frac{Q}{4\pi\varepsilon_0 r_B} - \frac{Q}{4\pi\varepsilon_0 r_A}\right] = q(V_B - V_A) \qquad 3.8$$

This is the charge q being moved, multiplied by the potential difference between A and B.

If $V_B > V_A$ and the charge $q > 0$, $W > 0$

Example

You are given a fixed $-2\mu C$ charge.

a. Calculate the electrostatic potential due to a
   - (i) at point A 2 mm from the charge
   - (ii) at point B 5 mm from the charge

b. Calculate the potential difference
   - (i) from A to B
   - (ii) from B to A

c. Calculate the work done in moving
   - (i) a $-3\mu C$ from A to B
   - (ii) the charge in (i) back to B

   Explain your answer in each case.

a. $V(r) = \dfrac{Q}{4\pi\varepsilon_0 r}$

(i) $V(2\,mm) = \dfrac{9\times10^9 \times(-2\times10^{-6})}{2\times10^{-3}} = -9\times10^6 V$

(ii) $V(5\,mm) = \dfrac{9\times10^9 \times(-2\times10^{-6})}{5\times10^{-3}} = -3.6\times10^6 V$

b. Potential difference

(i) from A to B is $V_{A\to B} = V_B - V_A = -3.6\times10^6 - (-9\times10^6) = 5.4\times10^6 V$

(ii) from B to A is $V_{B\to A} = V_A - V_B = -9\times10^6 - (-3.6\times10^6) = -5.4\times10^6 V$

c. The work done in

(i) moving a $-3\mu C$ from A to B is $qV_{A\to B} = -3\times10^{-6} \times 5.4\times10^6 = -16.2\,J$

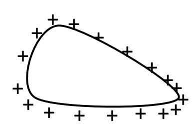
(negative work if moving it farther away, since there is mutual repulsion)

(ii) moving a $-3\mu C$ from B to A is $qV_{B\to A} = -3\times10^{-6} \times -5.4\times10^6 = 16.2\,J$

(positive work if moving it closer, since there is mutual repulsion)

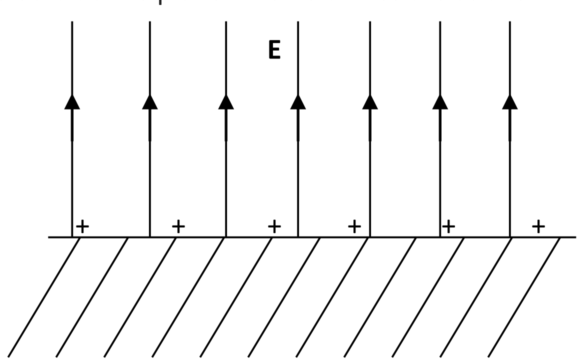
Arrangement of Charges on a Conductor

A charged solid spherical conductor has all the charges evenly distributed on the surface of the conductor. This is to ensure the minimum force of repulsion between the charges. This happens at electrostatic equilibrium, when the charges no longer move. This also means that there will be no component of the electric field parallel to the surface of the sphere, because were this not to be the case, the charges would move, contrary to the assumption that the system was in equilibrium. As such, field lines are perpendicular to the surface of conductors. For a conductor that is not spherical, the charges are not uniformly distributed on the surface. This is because the forces between them have two components, one parallel to the surface and one perpendicular to the surface. The ones perpendicular to the surface cancel out on opposite sides of the sharp edge. This means that the force pushing them away from each other on the surface is reduced. They tend to then cluster more together at pointed edges (Fig.).



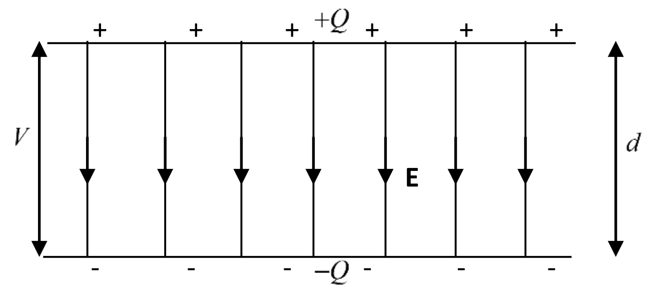
The Electric Field out of a Conducting Plane

We chose to bring this topic in to deepen your knowledge of electrostatics and introduce a physical system that produces a uniform field. Consider a charged conducting plane. There cannot be an electrostatic field inside a conductor if it is already at electrostatic equilibrium. This is because if there were to be such a field, then the electrons inside the conductor will move, negating electrostatic equilibrium. All the charges are outside the plane. The field lines are as shown in Fig. It is a uniform field.



The Electric Field between two Parallel Oppositely Charged Sheets

We consider two parallel sheets with equal and opposite charges as shown in Fig. . separated by a distance d



In this case, since the electric field is constant, it is given as

$$E = \frac{V}{d} \qquad 3.9$$

so that $V = Ed$, a linear function of the distance between the plates. We can take the negative plate as 0 potential. Then the upper plate will be at potential V. It is at a higher potential, which is why a positive charge will move from it to the lower plate at a lower potential. On the other hand, a negative charge will move from the plate at a lower potential to the plate at a higher potential. Another way of seeing this is to see the positive charge as being attracted by the lower negative plate and an electron being attracted by the upper positive plate. If you launch a charge at right angle to the field, it experiences a constant force

$$F = qE \qquad 3.10$$

which is perpendicular to its motion. This is like the situation with a projectile launched horizontally. Instead of the acceleration $a = -g$ in the case of the projectile in the field of the earth, the acceleration in this case is given by

$$F = qE = ma \Rightarrow a = \frac{qE}{m} \qquad 3.11$$

The charge moves on a parabola until it hits the plate to which it is directed.

Mechanical Energy of a Charge in an Electric Field

We have said it earlier that an electrostatic field is a conservative field. As such, the mechanical energy of a charge is conserved. Let us pick two positions and two velocities, r_A, r_B, v_A, v_B for a particular charge q and mass m moving in the electric field of charge Q. Then,

$$\frac{1}{2}mv_A{}^2 + \frac{qQ}{4\pi\,\varepsilon_0 r_A} = \frac{1}{2}mv_B{}^2 + \frac{qQ}{4\pi\,\varepsilon_0 r_B} \qquad 3.12$$

Hence,

$$\frac{1}{2}mv_A{}^2 - \frac{1}{2}mv_B{}^2 = \frac{qQ}{4\pi\,\varepsilon_0 r_B} - \frac{qQ}{4\pi\,\varepsilon_0 r_A} = q(V_B - V_A) \qquad 3.13$$

$$\frac{1}{2}mv_A{}^2 - \frac{1}{2}mv_B{}^2 = q(V_B - V_A) \qquad 3.14$$

The right hand side is the work done in moving charge from A to B. This is equal to a change in kinetic energy of the body. A loss in kinetic energy results in a gain in electrostatic potential energy. If q is positive, when it is repelled by the charge Q, it is going from a point of higher electrostatic potential energy to a point of lower electrostatic potential energy. This results in a loss of electrostatic potential energy. This loss in electrostatic potential energy results in an increase in kinetic energy, which manifests as the charge having a greater velocity. We say the charge has been accelerated by the field. Similarly, suppose you launch a positive charge q at the charge Q. Since both charges are positive, the charge q is going from a point of lower electrostatic potential energy to a region of higher electrostatic potential energy. It must slow down as it loses kinetic energy due to the increases in the electrostatic potential energy. The charge is decelerated.

Example

A potential difference of 20 V accelerates a charge $2\mu C$ from rest. Calculate the velocity of the charge if its mass is $10^{-2}\,kg$.

Position A: $v_A = 0, v_B = v, V_B < V_A \Rightarrow V_B - V_A < 0$ (the only way you can accelerate a positive charge in the desired direction. A positive potential difference will accelerate it in the opposite direction.

$$\frac{1}{2}mv_A{}^2 - \frac{1}{2}mv_B{}^2 = q(V_B - V_A)$$

$$\frac{1}{2}m\times 0^2 - \frac{1}{2}m\times v^2 = q(V_B - V_A) = -qV \Rightarrow v^2 = \frac{2qV}{m} = \frac{2\times 2\times 10^{-6}\times 20}{10^{-2}} = 8\times 10^{-3} \Rightarrow v = \sqrt{8\times 10^{-3}} = 0.089\,m/s$$

Explanation: The charge accelerated is positive. The acceleration can be achieved by a lower potential to draw it away from its initial position, by placing a negative charge in the direction the charge is to move. This is why V is negative.

You might circumvent all this by just applying the formula

$$qV = \frac{1}{2}mv^2 \Rightarrow v = \frac{2qV}{m}$$

bearing in mind that the velocity v must be positive.